This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2010 proceedings

A Channel Portfolio Optimization Framework for Trading in a Spectrum Secondary Market

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***Abstract*—State-of-the-art spectrum auctions are designed under a primary market paradigm to conduct spectrum trading between legacy owners and large cognitive service providers. In our previous work, we established a spectrum *secondary market* based on double auctions, and showed that it significantly improves spectrum utilization and user performance by allowing secondary users to dynamically trade among themselves their channel holdings obtained in the primary market. In this paper, we devise a channel portfolio optimization framework in order for users to make intelligent trading decisions without burdensome overhead. By viewing each channel in the secondary market as a stock, users assess its characteristics, and derive which channels to buy or sell at what price and quantity as a portfolio optimization problem to maximize the expected utility. Coupled with the robust secondary market design, the channel portfolio optimization framework offers salient performance with low complexity as corroborated in our simulations.**

I. Introduction

It is widely believed that current static spectrum assignment policy creates artificial spectrum scarcity in face of the proliferation of wireless technologies and devices. Along the line of dynamic spectrum access, spectrum auctions are perceived to be fair and efficient solutions of future spectrum trading where spectrum can be granted to those who value it most and can use it most efficiently [1]–[5]. Conventional spectrum auctions are proposed under a *primary market* paradigm. They are performed weekly or daily with legacy spectrum owners on the selling side and cognitive service providers on the buying side. Channels are often modeled to be homogeneous, and demands are assumed to be static. From an economics perspective, such an approach parallels a primary market of the capital market [6], and is only suitable to deal with the issuance of relatively long-term spectrum leases from legacy owners to large cognitive entities.

On the contrary, we mainly focus on dynamic spectrum trading among individual cognitive users themselves, *e.g.* mesh routers of small wireless networks, APs of home networks, etc. By shifting to a micro perspective, we observe that the underlying assumptions of the primary market paradigm no longer hold. For small users, traffic demand is extremely bursty as widely observed by existing works [7]. Moreover, channel bandwidth is of a finer granularity now, exhibiting significant time and frequency selectivity due to fading and user mobility as reported by extensive measurements [8]. The monolithic primary market paradigm becomes inherently inefficient, if not detrimental, when applied to this scenario.

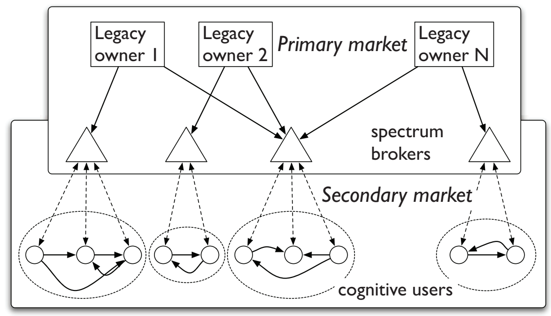


Fig. 1. The conceptual spectrum market structure for cognitive users.

In our previous work [9], [10], we pushed the state-of-the-art to the next level by going beyond a primary market. We established a novel spectrum *secondary market*. It coexists with the primary market through "spectrum brokers" as shown in Fig. 1. The primary market is the marketplace where spectrum brokers, with multiplexed demands across users of a certain area [1], bid for relatively long-term spectrum leases from legacy owners based on existing solutions [1]–[5]. The leased spectrum resources are then traded dynamically *amongst* cognitive users in the same area through the secondary market in a much finer time scale, to adapt to the time-varying demands and channel conditions. Towards this end, we devised a novel double auction mechanism, proved its truthfulness, asymptotic efficiency, budget-balance and individual rationality, and showed that it significantly improves spectrum utilization and user performance since secondary trading makes the spectrum more liquid and easier to obtain and relinquish [6].

With the spectrum secondary market, cognitive users have to make trading decisions on what channels to buy or sell, and at what prices and quantities, at the very start of each period of trading. Such trading decision making problem in an informationally decentralized and heterogeneous market environment has received little research attention so far. In the previous paper [10], we showed that the seemingly formidable problem can be tackled by a reinforcement learning framework, which essentially adopts a systematic trial-and-error way to derive the optimal decision policy. However, the learning algorithm requires a large amount of data to be calibrated, which may take many rounds of trading. The system-wide performance is inevitably scarified especially before the learning algorithm converges. To conquer this challenge, we let each user simulate hypothetical interactions with the market after each real interaction, and update its trading policy with the simulated data. Though the convergence problem is alleviated, extra computational overhead is also introduced. Moreover, hypothetical interactions assume the market can be modeled by a stationary Markov process, which may not be the case for dynamic spectrum trading.

In this work, we seek alternative lightweight solution to the trading decision making problem in order to improve the performance. We apply finance theory into spectrum secondary trading, and propose a novel channel portfolio optimization framework. To each user, each channel is viewed as a stock with time-varying price and risk. Its reservation price, *i.e.* the highest (lowest) price a user is willing to buy (sell) it, equals the utility it can bring (take) to (from) the user according to the channel condition. Risk can be roughly defined as the covariance of the throughput of channels. Since the channel conditions may fluctuate within a trading period, especially in scenarios with deep fading and high mobility, users' rationale in making the trading decision is to maximize their utilities determined by the expected throughput and funds available after trading, subject to budget and risk constraints. The risk constraint is essentially a quality-of-service constraint on the maximum level of throughput variance of the channel portfolio. Through extensive simulation studies we show that the portfolio optimization framework provides satisfactory performance while alleviates the overhead issue and slow convergence of the learning based solution.

II. The Spectrum Secondary Market

We start by introducing the spectrum secondary market established in our previous work [9]. We consider a micro-level cognitive radio network covered by one spectrum broker, with many cognitive users using possibly different technologies. The only assumption about users is that they use OFDMA as recommended by the IEEE 802.22 draft [11] for cognitive radio networks. OFDMA has already been implemented in various technologies including IEEE 802.16, 802.20, LTE and etc. We assume fading between OFDM subchannels far away from each other is uncorrelated, and each subcarrier of the same subchannel has the same fading statistics. One subcarrier is then the smallest trading unit.

The secondary market institution is a periodic double auction with multiple divisible *commodities*, *i.e.* subchannels. In each period of trading, for an arbitrary subchannel, there are a number of buyers and sellers willing to trade. Multi-unit bids and asks are submitted to the spectrum broker serving as the auctioneer. Then the winner and payment determination algorithms [9] are applied to determine the winning bids and asks, match the total supply with demand, set the transaction price, and calculate the payment for each winning user. These algorithms are designed to support multi-unit trading, and can be rigorously proven to enforce several desired economic properties. For example, *truthfulness* can be proved such that no user can expect a higher utility gain by setting its price different from the true valuation of the subchannel, *i.e.* the reservation price. Meanwhile, *asymptotic efficiency* and *individual rationality* can be achieved, which means the mechanism maximizes the social welfare when the number of users goes to infinity, and for the winning users, the expected utility gain is guaranteed to be non-negative, respectively.

III. A Channel Portfolio Optimization Framework

In this section, we introduce the channel portfolio optimization framework inspired by finance theory in details.

A. Channels as Stocks: Models

We assume that the network operates in a slow fading environment, so that channel estimation is possible. This can be done on a per-frame basis using SNR or BER directly from the physical layer. The achievable throughput can then be obtained by adapting the channel coding and modulation schemes to the channel condition with some rate adaptation algorithm, such as the one in [12]. Notice that the length of the trading interval is decided according to channel dynamics and communication overhead considerations, and can be on the order of tens of frames or packet transmissions [13]. Hence, the channel quality fluctuates within one trading interval, and thus the mean achievable throughput of the previous trading interval is used for the trading decision making at the current interval, which is valid by the slow fading assumption. Finally we comment that such channel estimation is feasible and accurate, and incurs little overhead when implemented with various physical layer technologies, as suggested by existing studies [12].

With the above assumptions, we can view the wireless channels as *stocks* with different reservation prices and risks in the market analogous to the stock market with multiple buyers and sellers. Due to the channel and user diversity, each individual user has an independent perception of a particular channel. Therefore, to address the trading decision making problem, our portfolio optimization framework naturally needs to answer the following two questions: 1) How does a user price each channel, *i.e.* assess the reservation price and risk based on the expected throughput? 2) How does a user decide which channels to buy, or to sell, and at what quantities?

B. Pricing the Channel

In our market, the consistent objective of any selfish user is to maximize its expected long-run self-interest, which can be represented by a utility function. Hence, the reservation price of a channel is directly related to the marginal utility gain it bears. The utility function needs to be quasi-linear to ensure that users can compensate each other with payments [6]. As such, in our problem, the utility function includes the benefit of satisfying traffic demand, which is non-linear, as well as the amount of funds that can potentially be used to purchase more spectrum, which is the linear part.

For trading interval t, let $\mathbf{X_i}(t)$ denote the vector of channel holdings for user i after trading, and $\mathbf{R_i}(t)$ denote its expected throughput on all channels. $B_i(t)$ represents the amount of funds it possesses after trading and $D_i(t)$ denotes the traffic demand. The utility function can then be expressed as:

$$U_i\left(\mathbf{X_i}(t), B_i(t)\right) = \epsilon_i \min\left\{\frac{\left(\mathbf{X_i}(t)\right)^{\mathsf{T}}\mathbf{R_i}(t)}{D_i(t)}, 1\right\} + B_i(t),$$

where ϵ_i is a positive parameter that indicates the relative importance of the current demand satisfaction, in comparison with the future trading potential. We assume that all users have the same form of utility functions, but they may have different ϵ's that are only privately known to characterize their preferences. Such definition of utility function motivates users to trade among themselves in order to improve their utilities dynamically.

With the utility function defined, the reservation price of buying a unit of a channel is readily obtained as follows:

$$P^b_{i,c}(t) = \epsilon_i \left(\min\left\{\frac{\left(\mathbf{X_i}(t-1)\right)^{\mathsf{T}}\mathbf{R_i}(t) + R^c_i(t)}{D_i(t)}, 1\right\} - \min\left\{\frac{\left(\mathbf{X_i}(t-1)\right)^{\mathsf{T}}\mathbf{R_i}(t)}{D_i(t)}, 1\right\}\right) \quad (1)$$

where $\mathbf{X_i}(t-1)$ is the channel portfolio before trading at t. Likewise the reservation price of selling a channel can be defined as the utility loss it will cause.

The risk of using the channels arises from the fact that the channel conditions change within one trading interval. As finance theory suggests [6], it may be represented by the covariance matrix of the instantaneous throughput across channels, which can be obtained by the channel estimation algorithm.

C. What to Trade, and How Many?

To address the second question on which channels to trade and at what quantities, we argue that the user rationale can be summarized as an optimization problem. A given user seeks to optimize its channel portfolio so as to maximize the total expected utility after trading, subject to a budget constraint. Meanwhile, it also tries to control the variance of the expected throughput of the portfolio to maintain a certain level of quality of service, which can be calculated from the covariance matrix.

Without ambiguity, we drop the time index t, and use the superscript $(\cdot)^-$ to denote a quantity at $t-1$ in the sequel. Let $\mathbf{C_i}$ denote user i's covariance matrix of throughput, $\mathbf{P_i}$ its reservation price vector, and θ_i the tolerance threshold of the throughput variance respectively. Expressed succinctly, the channel portfolio optimization problem is:

$$\max_{\mathbf{X_i}} \ \epsilon_i \min\left\{\frac{\mathbf{X_i}^{\mathsf{T}}\mathbf{R_i}}{D_i}, 1\right\} + E(B_i) \quad (2)$$

$$\text{s.t. } \mathbf{X_i}^{\mathsf{T}}\mathbf{P_i} - (\mathbf{X_i^-})^{\mathsf{T}}\mathbf{P_i} \le B_i^-, \quad (3)$$

$$\mathbf{X_i}^{\mathsf{T}}\mathbf{C_i}\mathbf{X_i} \le \theta_i, \quad (4)$$

(3) represents the budget constraint. Since our mechanism is individual rational, the reservation price P^c_i is a worse-case estimation of the transaction price P^c if user i is a winner of the auction for c [9]. Thus, the left side of (3) denotes the *maximum* net funds needed to acquire the optimal portfolio given the current one. It can be negative if the total proceeds from selling exceed the total costs of buying, and must be no larger than the total wealth that user i possesses before trading, *i.e.* B_i^-.

To calculate the expectation of funds available after trading, we first notice that $E(B_i) = B_i^- - (\mathbf{X_i} - \mathbf{X_i^-})^{\mathsf{T}} \cdot E(\mathbf{P})$, where $\mathbf{P}$ is the transaction price vector. Hence user i has to estimate the transaction prices based on what it has observed up to the previous trading period. By the prevalent *efficient market hypothesis* in financial economics, we assume here that the transaction price process is a *martingale*, and therefore $E(\mathbf{P}) = \mathbf{P}^-$, where $\mathbf{P}^-$ is the transaction price in the previous trading interval [6]. Thus the portfolio optimization problem can be alternatively formulated as follows:

$$\textit{OPT:} \quad \max_{\mathbf{X_i}} \ \epsilon_i \min\left\{\frac{\mathbf{X_i}^{\mathsf{T}}\mathbf{R_i}}{D_i}, 1\right\} - \mathbf{X_i}^{\mathsf{T}}\mathbf{P}^-$$
$$\text{s.t. } (3), (4).$$

D. Deriving the Optimal Portfolio

By observing the objective function of *OPT*, clearly we can see that increasing $\mathbf{X_i}$ further after the demand is fully satisfied will decrease the utility. Hence, we may simplify *OPT* without the minimization operator:

$$\textit{OPT\_S:} \quad \max_{\mathbf{X_i}} \ \mathbf{X_i}^{\mathsf{T}}(\mathbf{A_i} - \mathbf{P}^-)$$
$$\text{s.t. } \mathbf{X_i}^{\mathsf{T}}\mathbf{R_i} \le D_i,$$
$$\text{and } (3), (4),$$

where $\mathbf{A_i} = \epsilon_i D_i / \mathbf{R_i}$.

To solve *OPT_S*, we notice that it is an integer program and is NP-hard in general. We relax the integer constraint and let X^i_c be a non-negative real number. Then it becomes a convex program that can be solved in its dual domain. Introduce Lagrangian multipliers λ, μ, ν and the dual problem can be written as:

$$\min_{\lambda,\mu,\nu} \ g(\lambda, \mu, \nu) \quad (5)$$
$$\text{s.t. } \lambda, \mu, \nu \ge 0,$$

where

$$g(\lambda,\mu,\nu) = \max_{\mathbf{X_i}} \ \mathbf{X_i}^{\mathsf{T}}\left(\mathbf{A_i} - \mathbf{P}^-\right) + \lambda\left(B_i^- - \left(\mathbf{X_i} - \mathbf{X_i^-}\right)^{\mathsf{T}}\mathbf{P_i}\right) + \mu\left(\theta_i - \mathbf{X_i}^{\mathsf{T}}\mathbf{C_i}\mathbf{X_i}\right) + \nu\left(D_i - \mathbf{X_i}^{\mathsf{T}}\mathbf{R_i}\right).$$

To solve $g(\lambda, \mu, \nu)$, note that

$$\frac{\partial g(\lambda,\mu,\nu)}{\partial \mathbf{X_i}} = \mathbf{A_i} - \mathbf{P}^- - \lambda\mathbf{P_i} - 2\mu\mathbf{C_i}\mathbf{X_i} - \nu\mathbf{R_i}.$$

By KKT conditions, $\frac{\partial g(\lambda,\mu,\nu)}{\partial \mathbf{X_i}} = 0$. Thus, the optimal $\mathbf{X_i}$ for $g(\lambda, \mu, \nu)$ is

$$\mathbf{X_i^*}(\lambda,\mu,\nu) = \frac{1}{2\mu}\mathbf{C_i}^{-1}\left(\mathbf{A_i} - \mathbf{P}^- - \lambda\mathbf{P_i} - \nu\mathbf{R_i}\right). \quad (6)$$

Since the relaxed version of *OPT_S* has zero duality gap, it can be solved optimally by solving its dual problem. Subgradient methods can be used here to iteratively search for the optimal dual variables with which the optimal primal variables can be easily recovered. Finally we round the fractional channel allocation vector $\mathbf{X_i^*}$ to the floor of each element to conform to the integer constraint. The difference between the resulting optimal channel portfolio and the current portfolio, *i.e.* $\tilde{\mathbf{X}}_\mathbf{i} - \mathbf{X_i^-}$, is the trading quantity vector which specifies how many units user i wants to buy, if the difference is positive, or to sell if otherwise, for each channel. Together with the reservation price vector $\mathbf{P_i}$ determined according to (1), user i makes the trading decisions, forms its bids and asks for all channels and submits to the auctioneer. The complete algorithm of the trading decision making based on channel portfolio optimization is summarized as follows.

Algorithm 1 *Decision Making Algorithm based on Channel Portfolio Optimization.*

1. Each user i periodically runs a channel estimation algorithm, such as the one in [12], between two tradings at $t-1$ and t to obtain the mean throughput vector $\mathbf{R_i}$ and the covariance matrix $\mathbf{C_i}$.
2. At t, i determines its reservation price vector $\mathbf{P_i}$ according to (1), and solves the channel portfolio optimization problem *OPT_S* as follows.
   1) Initialize $\lambda^{(0)}, \mu^{(0)}, \nu^{(0)}$.
   2) Given $\lambda^{(k)}, \mu^{(k)}, \nu^{(k)}$, solve $g(\lambda, \mu, \nu)$ according to (6).
   3) Perform subgradient updates for λ, μ, ν, where $\vartheta_1, \vartheta_2, \vartheta_3$ follow a diminishing step size rule:
   $$\lambda^{(k+1)} = \left[\lambda^{(k)} - \vartheta_1^{(k)} \left(B_i^- - \left(\tilde{\mathbf{X}}_\mathbf{i} - \mathbf{X_i^-}\right)^\mathsf{T} \mathbf{P_i}\right)\right]^+$$
   $$\mu^{(k+1)} = \left[\mu^{(k)} - \vartheta_2^{(k)} \left(\theta_i - \mathbf{X_i}^\mathsf{T}\mathbf{C_i}\mathbf{X_i}\right)\right]^+$$
   $$\nu^{(k+1)} = \left[\nu^{(k)} - \vartheta_3^{(k)} \left(D_i - \mathbf{X_i}^\mathsf{T}\mathbf{R_i}\right)\right]^+$$
   4) Return to step 2) until convergence.
3. Submit the bids and asks formed by the trading quantity vector $\tilde{\mathbf{X}}_\mathbf{i} - \mathbf{X_i^-}$ and the reservation price $\mathbf{P_i}$.

E. Discussions

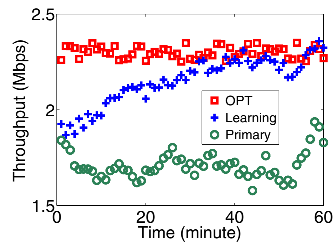
Highly efficient channel estimation algorithms are commonly available in the literature as we discussed before [12]. For the subgradient method, its complexity is polynomial in the dimension of the problem, which is 3 for $g(\lambda, \mu, \nu)$. Hence, the complete algorithm has low complexity.

In solving *OPT_S* we have relaxed the integer constraint, which introduces integrality gap. Characterizing the integrality gap of our rounding-based solution may be possible and can be one of the future work. Through simulation studies we observe that the rounding-based algorithm provides satisfactory performance, which justifies its use here to approximate the optimal solution.

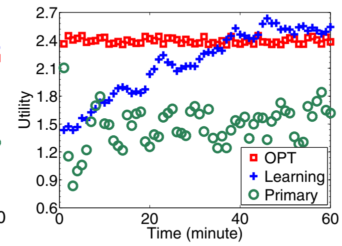
IV. Simulation Results

We are now ready to resort to extensive simulations to study the performance of our portfolio optimization based algorithm. As no previous work has been done for the spectrum secondary market, we rely on the double auction in [5] as our performance benchmark, which represents state-of-the-art spectrum allocation in the primary market paradigm. Be reminded that the double auction in [5] only supports homogeneous channels and single-unit bids and asks, and therefore bidding and asking prices are randomly generated.

A. Simulation Settings



(a) Throughput



(b) Spectrum Utilization

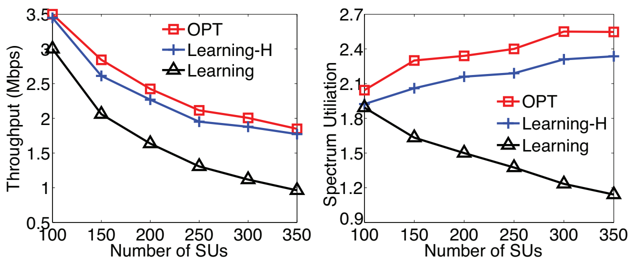
Fig. 2. Performance of secondary market and primary market.

We use practical settings of an OFDMA cognitive radio network, including channel frequency, bandwidth, and adaptive modulation and coding schemes, as specified in the IEEE 802.22 draft [11]. There are 48 channels, each containing 128 orthogonal subcarriers. Channel gain can be decomposed into a large-scale log normal shadowing component with standard deviation of 5.8 and path loss exponent of 4 and a small-scale Rayleigh fading component. The frequency selectivity is characterized by an exponential power delay profile with a delay spread 1.257μs. The time selectivity is captured by the Doppler spread, which depends on the user's speed. We assume every user moves around the network area according to the random waypoint model with its speeds (in km/h) following a uniform distribution $\mathrm{U}[0, 10]$. The combined complex gain is generated using an improved Jakes-like method [14].

We assume that data packets arrive at users following an asymptotically self-similar model, the ARIMA process, to model the bursty traffic [7]. All packets have the same size. The buffer is assumed to be sufficiently large, and the amount of data in it reflects user's demand. Two metrics are used to evaluate the performance: *(1)* Average User Throughput. *(2)* Spectrum Utilization as the average utility from all users.

B. Overall Performance

We first evaluate the effectiveness of our channel portfolio optimization algorithm. The simulation is performed for 60 minutes with 100 secondary users and 48 subchannels. Fig. 2 shows the results. We observe that the portfolio optimization algorithm denoted as "OPT" outperforms the learning algorithm by 20% before it converges. The performance margin becomes smaller as time goes, indicating the improved trading policy by the iterative learning algorithm. After convergence, "OPT" provides a similar level of performance as "Learning", and



(a) Throughput (b) Spectrum Utilization

Fig. 3. Investigation of the impact of market size.

enjoys a 30% throughput gain and a 35% spectrum utilization gain over conventional primary market based approach. These results demonstrate the advantage of the portfolio optimization framework since it does not require any training, and does not suffer from the slow convergence. The results also verify that with the secondary market, every channel is traded as a different stock with dynamic prices across users, and is efficiently utilized as time goes by, despite the temporal and spatial variation of user demands and link qualities.

C. Impact of Market Size

We investigate the impact of the market size in this section. Intuitively, a larger market involving a larger number of users provides more trading opportunities, and is therefore more efficient. This is also rigorously proved in our technical report [10]. However, increasing the size of the market has a negative impact on the learning algorithm, as the interactions with the market become more complex and uncertain, and it takes a longer period of training to converge. This may impair the user performance and overall spectrum utilization. Noticeably, the channel portfolio optimization framework does not suffer from this problem, as each user is only concerned about the characteristics of the channels that are independent from the market. To validate these intuitions, we perform simulations for 60 minutes under the same settings as in the previous section, but with a varying number of users.

Fig. 3 shows the results. We observe that, with respect to throughput, both "OPT" and "Learning" result in a decreasing trend as the size of the market increases. The reason is that each user has less resources to utilize on average with a fixed amount of total spectrum bandwidth. Moreover, the performance of "Learning" decreases more sharply, indicating the ill effect of slower convergence due to the increased level of market complexity. To remedy this problem we proposed using hypothetical interactions in [10], which we also evaluate as denoted by "Learning-H" in the figures. Clearly with improved convergence the throughput performance is largely uplifted, but it is still inferior to that of "OPT". Further, hypothetical interactions introduce extra computational overhead, and may be inaccurate in highly dynamic environments. These results coincide our intuition that the portfolio optimization framework greatly outperforms the learning solution in highly complex market environments.

With respect to spectrum utilization, surprisingly we observe that "OPT" and "Learning-H" perform better when the market expands. This implies that although each user has less resources on average, they are more efficiently allocated to users that can better utilize them in a larger market. In other words, this shows the increased efficiency of the market suggested by its asymptotic efficiency result. Also, we see that "Learning" is unable to harvest the increased market efficiency, again due to the impairing effect of severely slower convergence. Finally, "OPT" still outperforms both of the learning based solutions, verifying its effectiveness and robustness.

V. Concluding Remarks

In our previous work [9], [10], we presented a spectrum secondary market based on dynamic double auctions, which makes it possible for users to bilaterally trade their channel holdings. In this work, we devised a novel algorithm to solve the trading decision making problem based on a portfolio optimization framework that is widely used in finance. In our framework, each channel is viewed as a unique stock with dynamic characteristics that each user keeps track of. Then at each trading period, an optimization problem is efficiently solved to maximize the utility of the channel portfolio with budget and quality-of-service constraints. Simulation results corroborate the effectiveness of the algorithm in providing robust and good performance in dynamic environments while remedying the convergence issue of the learning solution we used before.

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