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XOR-Assisted Cooperative Diversity in OFDMA Wireless Networks: Optimization Framework and Approximation Algorithms

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***Abstract*—Network coding has been leveraged with cooperative diversity to improve performance in single channel wireless networks. However, it is not clear how network coding based cooperative diversity can be exploited effectively in multi-channel networks where overhearing is not readily available. Moreover, the question of how to practically realize the promising gains available, including multi-user diversity, cooperative diversity and network coding in multi-channel networks, also remains unexplored. This work represents the first attempt to unravel these two questions. In this paper, we propose XOR-CD, a novel XOR-assisted cooperative diversity scheme in OFDMA wireless networks. It can greatly improve the relay efficiency by over 100% mostly, thus uplifting the throughput performance by over 30% compared to conventional cooperative diversity scheme. In addition, we formulate a unifying optimization framework that jointly considers relay assignment, relay strategy selection, channel assignment and power allocation to reap different forms of gains. We design efficient polynomial time algorithms to solve the NP-hard problem with provably the best approximation factor, and verify their effectiveness using realistic simulations.**

I. Introduction

Recent years have witnessed a significant amount of research in the field of cooperative diversity. In a wireless network where a relay or a group of relays are located between the source and destination, a relay can facilitate the transmission by employing Amplify-and-Forward (AF) or Decode-and-Forward strategy (DF). In both cases, the combination of the directly transmitted signal and relayed signal provides a form of cooperative diversity, which has been shown to improve the throughput and/or power efficiency [1]–[3].

Network coding, another interesting technique to allow coding capability in exchange for network capacity gain, has spurred a plethora of research attention. It is shown to improve performance in multi-hop wireless networks [4] and information exchange paradigm [5], when used beyond traditional routing. In the context of cooperative diversity, network coding has also been leveraged at the relay to mix packets from different cooperative sessions, provided that the relay overhears and successfully decodes multiple transmissions and these transmissions share a common destination [6]–[8].

In this paper, we investigate the use of network coding in cooperative diversity from a new perspective. Previous work relies heavily on the single shared channel model which lays down the groundwork for the overhearing ability. We instead seek to understand network coding aided cooperative diversity in multi-channel networks, which has not yet been touched upon. We assume the context of Orthogonal Frequency Division Multiple Access (OFDMA) [9] based networks. Multi-channel networks impose unique challenges of using network coding aided cooperative diversity. In these networks, overhearing is no longer naturally available. Users can hear each other only when tuned to the same channel. Coding opportunities are therefore to be carefully *invented* and *engineered*, rather than opportunistically *harvested*. Moreover, network coding entails that the broadcast rate is confined to the worst rate of all links involved, aggravating the task of finding profitable coding opportunities.

Our *first* contribution in this paper is a novel XOR-assisted cooperative diversity scheme in OFDMA wireless networks, referred to as XOR-CD. It exploits coding opportunities on bi-directional traffic on the uplink and downlink of a Mobile Station (MS). Bi-directional traffic is profoundly available in OFDMA cellular networks, providing abundant network coding opportunities. Further, the channel conditions of the uplink and downlink of the same MS do not differ substantially, especially when path loss is dominant over random fading. This remedies the shortcoming of coding to the lowest rate of the spatially apart links that may differ significantly [10].

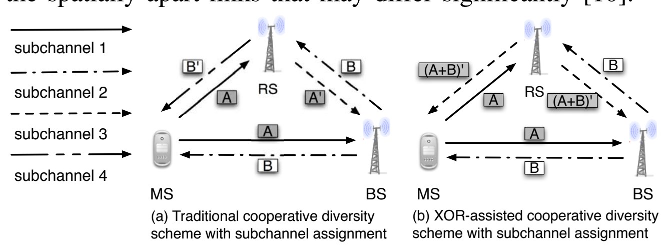

Fig. 1. The motivating scenario for XOR-assisted cooperative diversity in OFDMA wireless networks. 4 orthogonal subchannels are needed for conventional DF. For MS(BS), subchannel 1(2) is used to transmit the original packet $A(B)$ and subchannel 3(4) is used to transmit the re-encoded packet $A'(B')$ from RS, possibly with a different combination of channel coding and modulation scheme. With XOR, only 3 subchannels are needed resulting in a significant resource efficiency gain.

Fig. 1 illustrates an example to show the basic idea of XOR-assisted cooperative diversity (XOR-CD). Bi-directional traffic exists between MS and the BS. Assume that the Relay Station (RS) helps through cooperative relaying using orthogonal channels. By performing maximum ratio combining (MRC) over the two received copies of the same content, both MS and BS can enjoy a lower outage probability and thus higher

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throughput [3]. XOR network coding can be used here to mix packets A and B at the RS and multicast to MS and BS an re-encoded packet $(A \oplus B)'$ using only one single subchannel. Assume that channel coding and modulation are linear, $(A \oplus B)' = A' \oplus B'$. The MS and BS can still receive the intended information by XORing the combined packet with one that is known *a priori* to itself. Therefore, MRC can still be performed and cooperative diversity can still be capitalized.

The benefits of XOR-CD are intuitive. In the ideal case where channels are symmetric, and the BS-RS and MS-RS channel qualities are the same, XOR-CD achieves the same transmission rate for both cooperative sessions involved with a saving of one subchannel and the power of one transmission compared to the conventional DF. The saved subchannel and power can be used to accommodate more cooperative sessions, thereby further improving the network throughput.

Our *second* contribution is a unifying optimization framework to practically reap the promising gains of XOR-CD in OFDMA networks with reasonable complexity. We note that three kinds of gains can be exploited: (i) *multi-user diversity gain*: for a given data/relay subchannel, different MS experience independent fading processes, allowing us to assign the subchannel to the MS with the largest channel gain; (ii) *cooperative diversity gain*: RS helps the intended receiver to combat fading and improve SNR through cooperative relaying; (iii) *network coding gain*: bi-directional traffic between BS and MS is amenable to network coding which is utilized at RS to make relaying more resource efficient, increasing the network capacity. In light of different forms of gains available, our framework jointly considers **r**elay assignment, relay-**s**trategy selection, and **s**ubchannel assignment for both MS and RS in a single cell (hereafter referred to as RSS-XOR problem). We prove that RSS-XOR is NP-hard, and propose efficient polynomial-time algorithms based on packing with constant approximation factor.

Finally, we also extend our model to consider power allocation among cooperative sessions the RS supports. We propose subgradient-based algorithms to solve the power allocation problem in the dual domain, and show that XOR-CD consistently performs better in power-limited settings.

The remainder of this paper is structured as follows. The next section summarizes related work, and Sec. III introduces our system models. In Sec. IV we formally present the RSS-XOR framework and its counterpart that utilizes conventional DF only (called NO-XOR), and extend both models for power allocation. In Sec. V, we present efficient algorithms to solve both problems. We conduct extensive simulations to verify the effectiveness of algorithms and performance of XOR-CD in Sec. VI. We finally give concluding remarks in Sec. VII.

II. Related Work

This paper builds on prior work on cooperative diversity, whose roots can be traced back to the relay channel model studied in [11]. The popularity of cooperative diversity is owed to [1]–[3] where different strategies such as AF and DF are developed. We also build on work on network coding introduced in [12]. It is shown in [4] that network coding can greatly improve throughput in wireless multi-hop networks, when combined with routing. A similar conclusion is made for the information exchange paradigm in which two nodes exchange data with each other via a relay [5], a scenario similar to ours. However, the distinction is very clear: in these prior work, cooperative diversity is not leveraged as a mechanism to combat fading, since the two nodes cannot directly communicate without the relay. In [8], a network coding based cooperative diversity scheme is proposed. It entails the overhearing assumption with the single shared channel model, while XOR-CD assumes a multi-channel setting.

TABLE I
Related studies to this paper.

	Coding-based diversity	Channel assignment	Relay strategy	Power allocation
This paper	√	√	√	√
[8]	√	×	×	×
[13]	×	×	×	√
[14]	×	×	√	√
[15]	×	√	×	√
[16]	×	√	√	√

Optimization in cooperative networks has been studied in some previous work as well [13]–[16]. [13] considers power allocation for a simple triangle network with one pair of source-destination and one relay. [14] considers multi-hop ad hoc networks and proposes a framework that jointly considers routing, relay selection and power allocation. [15], [16] considers OFDMA based networks and are most related to our work. [15] studies channel assignment and power allocation for multi-hop OFDMA networks. [16] proposes solutions for joint optimization of channel assignment, relay strategy selection and power allocation in OFDMA cellular networks based on conventional AF and DF.

Different from previous work on network coding aided cooperative diversity and optimization of cooperative networks (summarized in Table I), to our knowledge, this work represents the first attempt to study cooperative diversity in multi-channel networks with the use of network coding. We propose a novel diversity scheme with XOR that can dramatically improve the resource efficiency of relaying and thereby boost throughput performance. More importantly, we formulate an optimization framework that jointly considers network coding, channel assignment, relay strategy selection and power allocation, which has not yet been discussed.

III. System Models

In this section, we introduce the underlying system models for the RSS-XOR optimization framework.

A. Network Models

We consider a single-cell OFDMA wireless network. BS is communicating with each MS with bi-directional traffic. The system operates in FDD mode, meaning that the uplink and downlink of an MS are assigned orthogonal sets of data subchannels. A small number of RS are employed in the cell to provide cooperative diversity. They may help some MS for transmissions on some of their *data subchannels*, using *relay* *subchannels* from a relay channel pool orthogonal to the data channel pool. One relay subchannel is used to support only one data subchannel of the MS in conventional cooperative diversity (CD) scheme. In the case of XOR-CD, one relay subchannel is used to support two data subchannels, one for uplink and one for downlink, as we illustrated in Sec. I. We further assume that the BS and MS always have frames to send and the OFDM frames are synchronized. In this case, it can be conceived that cooperative transmission progresses in parallel with direct data transmission in the long run. DF is used as the conventional CD scheme.

B. Channel Models

We model the wireless fading environment by large scale path loss and shadowing, along with small scale frequency-selective Rayleigh fading. Fading between different subchannels are independent. The network operates in a slow fading environment, so that channel estimation is possible and full channel side-information (CSI) is available, which makes the optimization possible. Such assumptions about the fading environment are commonly used as in [13], [15], [16].

An equal amount of power is allocated for data and relay transmissions across all data and relay subchannels. In the extended models with relay power allocation, however, RS can adjust the power level for each of the relay subchannels they use in order to confine themselves to their power budget.

IV. An Optimization Framework for XOR-CD

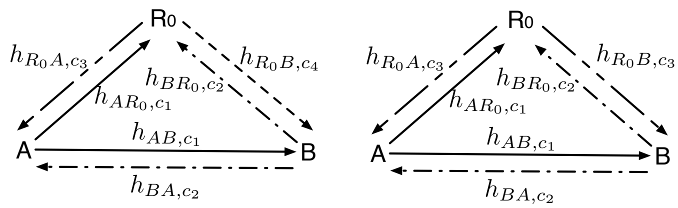
We present our optimization framework in this section.

A. Notations

Denote ζ, ψ, Ω and Φ as the set of data subchannels, relay subchannels, MS, and RS, respectively. $s \in \Omega$ denotes an MS and $r \in \Phi$ denotes an RS r, respectively. $l \in \mathcal{L}$ denotes a directed link from the source $S(l)$ to the destination $D(l)$ where $\mathcal{L}$ denotes the set of all links. Each link, being uplink or downlink, has a corresponding MS s such that $D(l) = s$ or $S(l) = s$. Let $M(l) = s$ denote this relationship between l and s. Each link can operate in one and *only* one of three modes, namely the direct transmission mode, conventional CD mode and XOR-CD mode, depending on the choice of relay strategy. Define function $R(c_i, l)$ as the achievable direct transmission rate of link l when it is assigned with subchannel $c_i \in \zeta$. For conventional CD, $R(c_i, c_r, r, p_l^{r,c_i,c_r}, l)$ is defined as achievable rate function of l, when RS $r \in \Phi$ is assigned to be the relay for transmission on data subchannel c_i, with allocated power p_l^{r,c_i,c_r} on relay subchannel c_r. For XOR-CD, $R(c_i, c_j, c_r, r, p_s^{r,c_i,c_j,c_r}, s)$ denotes the achievable rate function if r is the relay of s for its uplink transmission on c_i and downlink transmission on c_j, with allocated power p_s^{r,c_i,c_j,c_r} on relay subchannel c_r.

B. Information Theoretic Analysis

We first seek to provide an information theoretical analysis for the XOR-CD scheme in OFMDA wireless networks, in order to derive the rate functions for three transmission modes. The complex channel gains for different links are denoted as shown in Fig. 2. The noises are modeled as i.i.d. circularly symmetric complex Gaussian noises $\mathcal{CN}(0, N_0 W)$.



(a) Conventional DF (b) XOR-CD

Fig. 2. The channel models for DF and XOR-CD, where $h_{l,c}$ denotes channel gain of link l when it is assigned subchannel c.

1) Direct Transmission: For direct transmission, the achievable rate is found using the well-known formula (in b/s/Hz):

$$R(AB, c_1) = \log_2 \Big(1 + \frac{P \cdot |h_{AB,c_1}|^2}{\Gamma N_0 W}\Big), \qquad (1)$$

where Γ is the gap to capacity and P denotes the direct transmission power. For notational convenience we denote $\frac{|h_{AB,c_1}|^2}{\Gamma N_0 W}$ as CNR_{AB,c_1}, where CNR represents channel gain-to-noise ratio. Then the rate function can be expressed as:

$$R(AB, c_1) = \log_2(1 + P \cdot CNR_{AB,c_1}). \qquad (2)$$

2) Conventional CD: For the DF relay channel for transmission from A, first R_0 attempts to decode A's message. Assuming decoding is successful, R_0 transmits to B with power $p_{AB}^{R_0,c_1,c_4}$ using the relay subchannel c_4 as depicted in Fig. 2. Therefore, the maximum rate for this mode can be readily found to be

$$R(c_1, c_4, R_0, p_{AB}^{R_0,c_1,c_4}, AB) = \min\{\log_2(1+P \cdot CNR_{AR_0,c_1}), \log_2(1 + P \cdot CNR_{AB,c_1} + p_{AB}^{R_0,c_1,c_4} \cdot CNR_{R_0B,c_4})\}. \qquad (3)$$

Note that compared to result from [3], our result does not have $\frac{1}{2}$ before the expression. The reason is that a two-slot implementation is assumed in [3] with a shared channel, whereas we assume relay transmission progresses in parallel with data transmission, which is valid in the long run in our OFDMA-based multi-channel model.

Inspecting the rate function, we can see that increasing the relay power will first increase the rate, but not any more after reaching a threshold since pumping a higher rate will make the relay unable to decode. The threshold value of the relay power is such that

$$R(c_1, c_4, R_0, \tilde{p}_{AB}^{R_0,c_1,c_4}, AB) = \log_2(1 + P \cdot CNR_{AR_0,c_1}) = \log_2(1 + P \cdot CNR_{AB,c_1} + \tilde{p}_{AB}^{R_0,c_1,c_4} \cdot CNR_{R_0B,c_4}), \qquad (4)$$

which gives us

$$\tilde{p}_{AB}^{R_0,c_1,c_4} = \frac{CNR_{AR_0,c_1} - CNR_{AB,c_1}}{CNR_{R_0B,c_4}} P. \qquad (5)$$

Similar analysis can be given for $R(c_2, c_3, R_0, p_{BA}^{R_0,c_2,c_3}, BA)$ and the threshold power $\tilde{p}_{BA}^{R_0,c_2,c_3}$.

3) XOR-CD: The relay transmissions from R_0 to A and B are done by performing XOR over the two messages and multicasting using a single relay subchannel c_3. Therefore, the rate of this mode, for each of the two links involved, can be shown to be

$$\begin{aligned} &R(c_1, c_2, c_3, R_0, p_A^{R_0,c_1,c_2,c_3}, A) = \\ &\min\{\log_2(1 + P \cdot CNR_{AR_0,c_1}), \log_2(1 + P \cdot CNR_{BR_0,c_2}), \\ &\quad \log_2(1 + P \cdot CNR_{AB,c_1} + p_A^{R_0,c_1,c_2,c_3} \cdot CNR_{R_0B,c_3}), \\ &\quad \log_2(1 + P \cdot CNR_{BA,c_2} + p_A^{R_0,c_1,c_2,c_3} \cdot CNR_{R_0A,c_3})\}, \end{aligned} \quad (6)$$

assuming A is an MS and B is the BS. The first two terms in (6) represent the maximum rate at which the relay can reliably decode the source messages from both A and B, while the last two terms in (6) represent the maximum rate at which A and B can reliably decode the intended message given repeated transmissions from R_0's multicast. Again the threshold value of relay power at R_0 is such that

$$\begin{aligned} &R(c_1, c_2, c_3, R_0, \tilde{p}_A^{R_0,c_1,c_2,c_3}, A) = \\ &\min\{\log_2(1+P\cdot CNR_{AR_0,c_1}), \log_2(1+P\cdot CNR_{BR_0,c_2})\}. \end{aligned}$$

The expression for threshold relay power $\tilde{p}_A^{R_0,c_1,c_2,c_3}$ can then be derived. We omit details due to space constraints.

C. RSS-XOR Problem

Our main objective is to optimize the strategies of assigning appropriate sets of relay subchannels to RS and data subchannels to MS, and pairing RS to data subchannels of MS with different choices of relay strategies, in order to maximize the aggregated throughput subject to a fairness model. We consider proportional fairness model, given its ability to strike a good balance between utilization and fairness and its robustness with respect to changes in topology and power constraints [17]. Denote the throughput of link l as λ_l, then the *long-term* objective function under proportional fairness model can be expressed as $\max \sum_l \ln \bar{\lambda}_l$, where $\bar{\lambda}_l$ is the long-term average throughput for link l. The optimization needs to be done periodically as MS may move and channel qualities change over time. It is proved in [17] that the instantaneous optimization maximizing the marginal utility $\sum_l U_l'(\bar{\lambda}_l) \cdot \lambda_l$ at each epoch (interval) leads to a long-term maximization of $\sum_l U_l(\bar{\lambda}_l)$, if we assume link l has a concave utility function $U_l(\bar{\lambda}_l)$. Therefore, our objective at each epoch t is:

$$\max \sum_{l \in \mathcal{L}} \frac{\lambda_l}{\bar{\lambda}_l(t)} \quad (7)$$

For both uplink and downlink, traffic can be classified into three classes corresponding to three transmission modes, namely direct traffic, conventional CD traffic and XOR-CD traffic. Introduce three 0–1 decision variables $x_l^{c_i}$, y_l^{r,c_i,c_r} and z_s^{r,c_i,c_j,c_r}. $x_l^{c_i}$ indicates whether link l is assigned data subchannel c_i for direct transmission. y_l^{r,c_i,c_r} indicates whether link l is operating in conventional CD mode with RS r and data-relay subchannel pair (c_i, c_r). Each MS may be assigned multiple such channel pairs depending on the instantaneous channel quality. z_s^{r,c_i,c_j,c_r} indicates whether MS s is assigned with RS r and relay subchannel c_r for its uplink on data subchannel c_i and downlink on c_j for XOR-CD.

Since an equal amount of power P is used for every direct and relay transmission, the throughput constraint of link l can be characterized as follows:

$$\begin{aligned} \lambda_l = &\sum_{c_i \in \zeta} R(c_i, l) x_l^{c_i} + \sum_{c_i \in \zeta, c_r \in \psi, r \in \Phi} R\big(c_i, c_r, r, P, l\big) y_l^{r,c_i,c_r} \\ &+ \sum_{c_i, c_j \in \zeta, c_r \in \psi, r \in \Phi} R(c_i, c_j, c_r, r, P, s) \cdot z_s^{r,c_i,c_j,c_r}, \\ &\text{where } s = M(l),\ \forall l \in \mathcal{L}. \end{aligned} \quad (8)$$

For data subchannels, we dictate that each data subchannel can only be assigned to one link which operates in one of the three modes. Therefore,

$$\begin{aligned} \sum_{l \in \mathcal{L}} \Big(x_l^{c_i} + \sum_{c_r \in \psi, r \in \Phi} y_l^{r,c_i,c_r}\Big) + &\sum_{s \in \Omega, r \in \Phi, c_j \in \zeta, c_r \in \psi} \Big(z_s^{r,c_i,c_j,c_r} \\ &+ z_s^{r,c_j,c_i,c_r}\Big) \le 1, \forall c_i \in \zeta, \end{aligned} \quad (9)$$

where the first term accounts for the possibility that c_i is assigned for direct and conventional CD modes, and the second term accounts for the possibility of XOR-CD mode. Notice that this constraint also implicitly takes into consideration that each link can only operate in one of the three modes.

Similarly, each relay subchannel can be assigned to only one cooperative session, be it conventional CD session or XOR-CD session. Therefore,

$$\sum_{l \in \mathcal{L}} \sum_{r \in \Phi} \sum_{c_i \in \zeta} y_l^{r,c_i,c_r} + \sum_{s \in \Omega} \sum_{r \in \Phi} \sum_{c_i \in \zeta} \sum_{c_j \in \zeta} z_s^{r,c_i,c_j,c_r} \le 1, \quad \forall c_r \in \psi. \quad (10)$$

Consequently, the RSS-XOR problem becomes a mixed-integer program, with the objective (7), subject to constraints (8), (9), and (10).

D. NO-XOR Problem

We also provide a framework for the joint optimization with only conventional cooperative diversity, *i.e.,* the NO-XOR problem. It is studied as a baseline comparison. The NO-XOR problem can be formulated in a similar way as the RSS-XOR problem, with z_s^{r,c_i,c_j,c_r} equal to zero for any $c_i, c_j \in \zeta$, $c_r \in \psi$, $s \in \Omega$, $r \in \Phi$. Formally,

$$\begin{aligned} \text{NO-XOR: } \max &\sum_{l \in \mathcal{L}} \frac{\lambda_l}{\bar{\lambda}_l(t)} \\ \text{s.t. } \lambda_l \le &\sum_{c_i \in \zeta} R(c_i, l) x_l^{c_i} \\ &+ \sum_{c_i \in \zeta} \sum_{c_r \in \psi} \sum_{r \in \Phi} R\big(c_i, c_r, r, P, l\big) y_l^{r,c_i,c_r}, \\ \sum_{l \in \mathcal{L}} x_l^{c_i} + &\sum_{l \in \mathcal{L}} \sum_{r \in \Phi} \sum_{c_r \in \psi} y_l^{r,c_i,c_r} \le 1, \forall c_i \in \zeta, \\ \sum_{l \in \mathcal{L}} \sum_{r \in \Phi} &\sum_{c_i \in \zeta} y_l^{r,c_i,c_r} \le 1, \forall c_r \in \psi. \end{aligned} \quad (11)$$

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E. Power Allocation

We extend the two models by considering the scenario wherein each RS has a power budget constraint. RS then have to allocate the right amount of power across all cooperative sessions so as to maximize marginal utility. Mathematically, the throughput constraints of both problems are updated by replacing $R(c_i, c_r, r, P, l)$ with $R(c_i, c_r, r, p_l^{r,c,c_r}, l)$ and $R(c_i, c_j, c_r, r, P, s)$ with $R(c_i, c_j, c_r, r, p_s^{r,c_i,c_j,c_r}, s)$ in (8) and (11). Moreover, the constraint that the total power used at the RS cannot exceed its power budget can be expressed as follows for the RSS-XOR problem:

$$\sum_{l\in\mathcal{L}}\sum_{c_i\in\zeta}\sum_{c_r\in\psi} p_l^{r,c_i,c_r} + \sum_{s\in\Omega}\sum_{c_i\in\zeta}\sum_{c_j\in\zeta}\sum_{c_r\in\psi} p_s^{r,c_i,c_j,c_r} \le P_r, \forall r \tag{12}$$

where P_r denotes the power budget of RS r. Power allocation version of RSS-XOR can be formulated by adding constraint (12) into the original formulation.

For NO-XOR, the power constraint is simply:

$$\sum_{l\in\mathcal{L}}\sum_{c_i\in\zeta}\sum_{c_r\in\psi} p_l^{r,c_i,c_r} \le P_r, \forall r \in \Phi. \tag{13}$$

The power allocation version of NO-XOR is similarly formulated by adding constraint (13) into (11).

V. Approximation Algorithms

Conventional approaches to solve mixed-integer problems, such as branch and bound, are computationally formidable. Our solution algorithms need to be called at each epoch, making the task of deriving efficient heuristic algorithms imperative. We propose algorithms that can be applied to the BS and RS in real OFDMA-based wireless networks for the RSS-XOR and NO-XOR problems. Specifically, we first prove that RSS-XOR is NP-hard and show it can be solved in polynomial-time with an approximation ratio of 1.5 using our algorithm. We then show that NO-XOR can be *optimally* solved by transforming to weighted bipartite matching. Finally we propose a subgradient algorithm to solve power allocation of the two problems.

A. A Set Packing Algorithm for RSS-XOR

Solving the seemingly prohibitive RSS-XOR problem hinges on transforming to a weighted set packing problem. We first establish the equivalence and prove the NP-hardness, then propose our algorithm with a constant approximation factor.

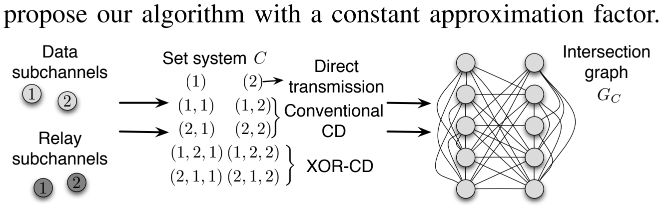


Fig. 3. Set construction and transformation into an intersection graph with 2 data subchannels and 2 relay subchannels. Vertices in G_C correspond to sets in C. Edges are added between vertices whose corresponding sets intersect.

Proposition 1: The RSS-XOR problem is equivalent to a maximum weighted 3-set packing problem, and is NP-hard.

Proof: Construct a collection of sets C from a base set $\zeta \cup \psi$ as shown in Fig. 3. There are three kinds of sets, representing three transmission modes respectively. (c_i), where $c_i \in \zeta$ represents the direct transmission mode with data subchannel c_i. (c_i, c_r) where $c_i \in \zeta, c_r \in \psi$ corresponds to the conventional CD mode with data subchannel c_i and relay subchannel c_r. The third kind, (c_i, c_j, c_r) corresponds to the XOR-CD mode with data subchannel pair (c_i, c_j) and relay subchannel c_r, where $c_i, c_j \in \zeta, c_r \in \psi$. Sets intersect if they share at least one common element, and are otherwise said to be disjoint. Each set has a corresponding weight, denoting the maximum marginal utility found across all possible assignments of this set to different combinations of RS and links. Specifically,

$$w_{(c_i)} = \max_{l} \frac{R(c_i, l)}{\bar{\lambda}_l(t)}, \tag{14}$$

$$w_{(c_i,c_r)} = \max_{l,r} \frac{R(c_i, c_r, r, P, l)}{\bar{\lambda}_l(t)}. \tag{15}$$

For (c_i, c_j, c_r), the weight is found over all possible assignments of this set to combinations of RS and uplink-downlink of an MS, since it can only be assigned to one MS. Formally,

$$w_{(c_i,c_j,c_r)} = \max_{s,r} \sum_{l:s=M(l)} \frac{R(c_i, c_j, c_r, r, P, s)}{\bar{\lambda}_l(t)}. \tag{16}$$

Note that $w_{(c_i,c_j,c_r)}$ essentially sums up rates of uplink and downlink of s since one XOR-CD session incorporates two cooperative transmissions. The optimization is to find the optimal strategy to choose the transmission mode and assign RS and channels to each link in order to maximize the aggregated throughput. The maximization is over all links. Equivalently, we can also interpret it as to find the optimal strategy to select disjoint channel combinations and assign RS and links to them so as to maximize the objective. In this alternative interpretation, the maximization is done over all possible channel sets by matching them to the best possible links and RS without duplicate use of channels. The solution found must exhaust all subchannels since we can always improve the total weight by adding sets corresponding to unassigned data and relay subchannels. The number of elements in a set is at most 3, therefore the problem is equivalent to weighted 3-set packing [18], which is NP-complete. ■

The assignment corresponding to the set weight is recorded down in a table T_{assign}. We see that, for sets (c_i), the size of weight search space is $|\mathcal{L}|$; for sets (c_i, c_r) and (c_i, c_j, c_r), the search space size is $|\Phi||\mathcal{L}|$. Thus, the weight construction process is of polynomial time complexity, given the number of three kinds of sets are also polynomials of $|\zeta|$ and $|\psi|$.

To propose a good approximation algorithm with reasonable time complexity, first we construct an intersection graph G_C of the set system C with set of vertices V_C and set of undirected edges E_C as shown in Fig. 3. Weighted set packing then can be generalized as a weighted independent set problem, the objective of which is to find a maximum weight subset of mutually non-adjacent vertices in G_C [19]. The size of sets is at most 3, therefore G_C is 3-claw free. Here a d-claw c is an induced subgraph that consists of an independent set T_c of d nodes. The best known approximation for the weighted independent set problem in claw-free graph is proposed in [19] and then acknowledged in [18], which we extend to form our algorithm *ALG2*.

First we introduce a greedy algorithm, called *Greedy*, that prepares the groundwork for *ALG2*. Define $N(K, L)$ to be the set of vertices in L that intersect with vertices in K, *i.e.* $N(K, L) = \{u \in L : \exists v \in K \text{ such that } \{u, v\} \in E \text{ or } u = v\}$. *Greedy* is a natural heuristic that repeatedly picks the heaviest vertex from among the remaining vertices and eliminate it and the adjacent vertices as shown below.

Algorithm 1 *Greedy*.

1. $S \leftarrow \varnothing$
2. **while** $V_C - N(S, V_C) \neq \varnothing$ **do**
3. choose $u \in V_C - N(S, V_C)$ with the maximum weight
4. $S \leftarrow S \cup \{u\}$
5. **end while**

A natural thought to improve on the maximal independent set found by *Greedy* is to do local search and replace a set with its claw with larger weight, which motivates *ALG2* summarized as in **Algorithm 2**. From [19], local improvements on the square of total weights solve the weighted independent set problem with a constant approximation factor of 1.5, which is the best result known so far [18]. Therefore we have the following:

Proposition 2: ALG2 provides at least $\frac{2}{3}$ of the optimum of RSS-XOR problem. This is the *best* performance guarantee one can have unless a better algorithm can be found for the weighted independent set problem.

Algorithm 2 *ALG2*.

1. Construct the collection of weighted sets C and transform into the weighted undirected graph G_C.
2. Obtain a maximal independent set S using *Greedy*.
3. **while** $\exists$ claw c such that T_c improves $w^2(S)$ **do**
4. $S \leftarrow S - N(T_c, S) \cup T_c$
5. **end while**
6. Assign channels to RS and links by searching the entries in T_{assign} corresponding to the sets present in S.

B. A Matching Algorithm for NO-XOR

We consider NO-XOR problem now. Surprisingly, we find that it can be optimally solved in polynomial time.

Proposition 3: The NO-XOR problem is equivalent to weighted bipartite matching over all data and relay channels, and can therefore be solved optimally.

Proof: Construct a bipartite graph $A = (V_1 \times V_2, E)$ where V_1 and V_2 correspond to the set of data subchannels ζ and relay subchannels ψ respectively, as shown in Fig. 4. We patch void vertices to V_2 to make $|V_2| = |V_1| = |\zeta|$. The edge set E corresponds to $|\zeta|^2$ edges connecting all possible pairs of channels in two vertex sets. Each edge (k, j) carries three attributes, (w_{kj}, l_{kj}, r_{kj}), where

$$w_{kj} = \max_{l,r} \frac{R(k, j, r, P, l)}{\bar{\lambda}_l(t)},$$
$$(l_{kj}, r_{kj}) = \arg\max_{l,r} \frac{R(k, j, r, P, l)}{\bar{\lambda}_l(t)}. \quad (17)$$

For edges connecting data subchannels to void relay subchannels that we patched, the edge weights become $(w_{kj}, l_{kj}, 0)$ where l_{kj} is the link providing maximum marginal utility if data subchannel k is used. This essentially captures the maximum marginal utility given by direct transmission.

Observe that A is bipartite, we can see the NO-XOR problem is equivalent to finding the maximum weighted bipartite matching on A. The second attribute of an edge (k, j) in the maximum matching represents the link assigned with this data-relay subchannel pair (k, j), while the third attribute dictates the transmission mode or the corresponding RS. A 0 in the third attribute simply means the link should work in direct transmission mode. Hence, the maximum matching found represents the comprehensive assignment of RS, data and relay subchannels, as well as the transmission strategy decision, therefore optimally solves the NO-XOR problem. ■

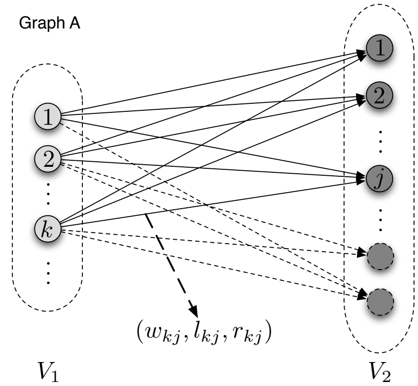


Fig. 4. The graphical model to show the equivalence of NO-XOR problem and weighted bipartite matching. Dotted vertices are void vertices patched. Not all links are shown here.

Several good polynomial-time algorithms exist for solving the bipartite matching problem, of which the Hungarian algorithm [20] is a popular choice. Since the graph construction is $O(|\zeta|^2 \cdot |\mathcal{L}| \cdot |\psi|)$, the whole algorithm is polynomial time.

C. A Power Allocation Algorithm

Finally, we turn our focus to the power allocation problem. The power limited versions of RSS-XOR and NO-XOR are proposed in Sec. IV-E. They are non-convex problems because of the integer constraints. Duality gap for non-convex problems is non-zero in general. However, in an OFDMA system with many narrow subchannels, the optimal solution of RSS-XOR and NO-XOR is always a convex function of P_r, because if two sets of throughputs using two different channel-RS-link assignments and relay strategies are achievable individually, their linear combination is also achievable by a frequency-division multiplexing of the two sets of strategies. This idea for non-convex problems of multi-carrier systems is discussed earlier in [21]. In particular, using the duality theory of [21], the following is true:

Proposition 4: The power limited RSS-XOR and NO-XOR problems have zero duality gap in the limit as the number of OFDM subchannels goes to infinity, even though the discrete selection of channels, RS and relay strategies are involved.

A detailed proof can be constructed along the same line of argument as in [21]. This proposition allows us to solve the non-convex problems in their dual domain. Note that although the proposition requires number of channels to go to infinity, in reality the duality gap is very close to zero as long as number of channels is large [16]. Consider the power limited RSS-XOR problem. The dual method for power limited NO-XOR problem can be developed in a similar way. Introduce an Lagrangian multiplier vector $\boldsymbol{\mu}$ to the power constraint (12) and the dual problem becomes

$$\min_{\boldsymbol{\mu}\geq 0} \; g(\boldsymbol{\mu}) \tag{18}$$

where

$$g(\boldsymbol{\mu}) = \max \sum_{l\in\mathcal{L}} \frac{\lambda_l}{\bar{\lambda}_l(t)} + \sum_{r\in\psi} \mu_r \Big(P_r - \sum_{l,c,c_r} p_l^{r,c,c_r} - \sum_{s,c_i,c_j,c_r} p_s^{r,c_i,c_j,c_r}\Big)$$
$$\text{subject to (8)–(10).} \tag{19}$$

Since each MS s corresponds to two links, we can equally split power used for XOR-CD p_s^{r,c_i,c_j,c_r} to these two links without violating the power constraint. Mathematically, we let

$$p_l^{r,c_i,c_j,c_r} = \begin{cases} \frac{1}{2} p_s^{r,c_i,c_j,c_r} & \text{if } s = M(l); \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The objective function of (19) can then be written as

$$\max \sum_l \Big(\frac{\lambda_l}{\bar{\lambda}_l(t)} - \sum_{r,c,c_r} \mu_r p_l^{r,c,c_r} - \sum_{r,c_i,c_j,c_r} \mu_r p_l^{r,c_i,c_j,c_r}\Big) + \sum_r \mu_r P_r. \tag{21}$$

$\sum_r \mu_r P_r$ is constant in (21) since $\boldsymbol{\mu}$ is given for each instance of $g(\boldsymbol{\mu})$. So solving the optimization (19) is equivalent to solving the following:

$$\max \sum_l \Big(\frac{\lambda_l}{\bar{\lambda}_l(t)} - \sum_{r,c,c_r} \mu_r p_l^{r,c,c_r} - \sum_{r,c_i,c_j,c_r} \mu_r p_l^{r,c_i,c_j,c_r}\Big)$$
$$\text{subject to (8)–(10).} \tag{22}$$

Compared with the original RSS-XOR problem, the only difference is the objective function which now includes the cost of power. $\boldsymbol{\mu}$ can be interpreted as a pricing variable vector for relay power. (22) can be thought of as maximizing the total marginal utility minus the total cost of relay power used, given the current prices of power at RS. This is easily decomposed into revenue maximization over every possible set of data and/or relay channels. Therefore, it can be solved using the approximation algorithm *ALG2* in Sec. V-A, with the weights being the maximum marginal revenue instead of the maximum marginal utility. For ease of exploration, we dictate that the relay power for each cooperative session is set to the threshold value as derived in Sec. IV-B2 and IV-B3. Then,

$$w_{(c_i,c_r)} = \max_{l,r} \frac{R(c_i,c_r,r,p_l^{r,c_i,c_r},l)}{\bar{\lambda}_l(t)} - \mu_r p_l^{r,c_i,c_r},$$
$$w_{(c_i,c_j,c_r)} = \max_{s,r} \sum_{l:s=M(l)} \frac{R(c_i,c_j,c_r,r,p_s^{r,c_i,c_j,c_r},s)}{\bar{\lambda}_l(t)} - \mu_r p_s^{r,c_i,c_j,c_r}.$$

After solving (22), the dual problem (18) can be readily solved via a subgradient method which repeatedly updates the power prices according to the demand/supply relationship at RS to regulate the power consumption. To summarize, the algorithm for solving the power limited RSS-XOR problem, referred to as *ALG3*, works as shown in **Algorithm** 3. Notice that the subgradient algorithm is suitable for distributed implementation across RS. Each RS is able to verify its power consumption, and update its own relay power price autonomously according to $\boldsymbol{\nu}^{(k)}$ informed by BS. The updated prices can be transmitted to the BS to solve the revenue maximization problem with a negligible amount of overhead.

Algorithm 3 *ALG3*.

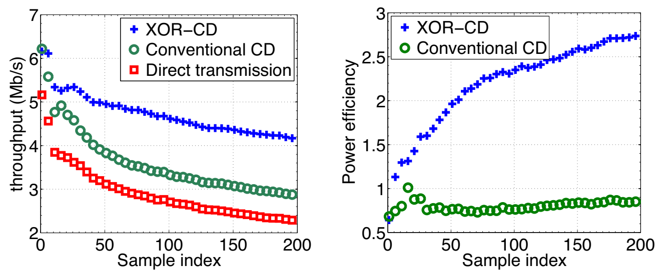
1. Initialize $\boldsymbol{\mu}^{(0)}$.
2. Given $\boldsymbol{\mu}^{(k)}$, solve the revenue maximization problem (22) using *ALG2* with weights being maximum revenue. Obtain the solution values $\hat{p}_l^{r,c,c_r}$ and $\hat{p}_l^{r,c_i,c_j,c_r}$, and the maximal independent set $\hat{S}$.
3. Perform a subgradient update for $\boldsymbol{\mu}$, where $\boldsymbol{\nu}^{(k)}$ follows a diminishing step size rule:
$$\mu_r^{(k+1)} = \Big[\mu_r^{(k)} - \nu_r^{(k)}\Big(P_r - \sum_{l,c,c_r} \hat{p}_l^{r,c,c_r} - \sum_{l,c_i,c_j,c_r} \hat{p}_l^{r,c_i,c_j,c_r}\Big)\Big]^+$$
4. Return to step 2 until convergence.
5. Assign channels to RS and links by searching the entries in T_{assign} corresponding to the sets present in $\hat{S}$ found in step 4.

VI. Performance Evaluation

A. Simulation Setup

The key of our experiment settings is to derive the achievable data rate of a subchannel when it is allocated to an arbitrary MS. This requires computing the corresponding SNR value. To generate realistic results, we adopt empirical parameters to model the wireless fading environment. The subchannel bandwidth is set to be 312.5 kHz. Data subchannels are centered around 5GHz, while relay subchannels are centered around 2.5 GHz. Channel gain between two nodes at each subchannel can be decomposed into a small-scale Rayleigh fading component and a large-scale log normal shadowing with standard deviation of 5.8 and path loss exponent of 4. The inherent frequency selective property is characterized by an exponential power delay profile with delay spread 15 μs. The time selective nature is captured by the Doppler spread, which depends on the MS's speed (throughout the simulation the MS are moving with speed uniformly distributed from 1 to 5 m/s according to random waypoint model with 0 pause period). The gap to capacity Γ is set to 1, which corresponds to perfect coding. The power constraint for each transmission is such that $\frac{P}{N_0 W} = 23$ dB. This corresponds to a medium SNR environment. Such an experimental setup is commonly used in related studies [15], [16].

B. Performance of XOR-CD



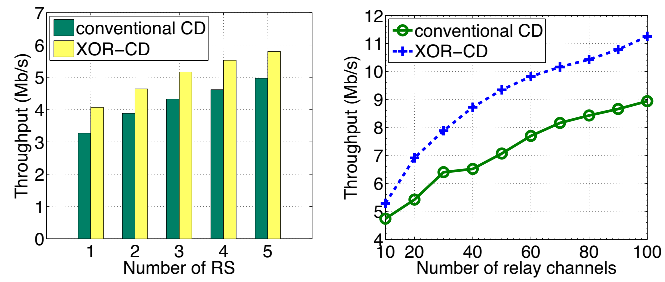
(a) Throughput comparison (b) Power efficiency comparison

Fig. 5. Performance of XOR-CD against conventional CD.

We first evaluate performance of XOR-CD compared with conventional CD and direct transmission. We focus on the scenario where 10 MS are uniformly located in a cell with 100m radius. To reduce the computation load we set the number of data subchannels to be 100, and that of relay subchannels to be 30. 1 RS is deployed in the cell. *ALG2* and *ALG3* as proposed in Sec. V-A and V-B are implemented to obtain the optimization results. Fig. 5(a) plots the time averaged throughput for a one-second period of time, with a sampling period of 5 ms. The optimization therefore is done for 200 times. We can see that the average throughput is slowly decreasing over time. The reason is that our objective takes fairness into account which makes the optimization favor a "slower" MS as time goes, resulting in a slowly decreasing trend. We can clearly see that XOR-CD outperforms the conventional diversity scheme by around 30%. This is as expected, because XOR-CD conserves relay channels that can be utilized to support more cooperative sessions. To further illustrate its superiority in this aspect, we study XOR-CD's relay resource efficiency. We define *power efficiency* to be the ratio of throughput improvement obtained over direct transmission, and the amount of relay power used. As we see in Fig. 5(b), XOR-CD's power efficiency is *significantly*, mostly over 100%, better than that of conventional CD. Finally, notice that the conventional diversity scheme alone provides over 20% improvement compared with simple direct transmission. This diversity gain is similar to the network coding gain, which further confirms the advantage of XOR-CD to "double" the diversity gain without any costs.

C. Effects of Relay Resources

Next, we study effects of relay resources. We focus on number of RS and number of relay subchannels. Fig. 6 shows the results. Intuitively, more RS provides a better chance for MS to find a nearby RS with better relay channel qualities. More relay subchannels enables more cooperative sessions to happen. Clearly, these two factors contribute to the increasing



(a) Effect of RS. Number of relay subchannels is 20. (b) Effects of relay subchannels. Number of RS is 3.

Fig. 6. Effects of Relay Resources. Number of data channels is 100, number of MS is 10, and cell radius is set to be 100m.

trend reflected in Fig. 6. XOR-CD consistently maintains a $20 - 30\%$ gain over conventional CD. We also note that the marginal gains are gradually diminishing for both factors. This can be explained as the optimization always tries to harvest the largest performance gains first. The diminishing trend suggests that we could use a small amount of relay resources to obtain a reasonably satisfactory improvement.

D. Effects of Path Loss

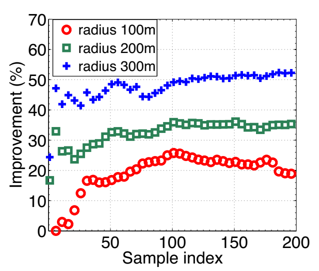


Fig. 7. Effects of path loss for 100 data subchannels, 30 relay subchannels, 10 MS and 3 RS.

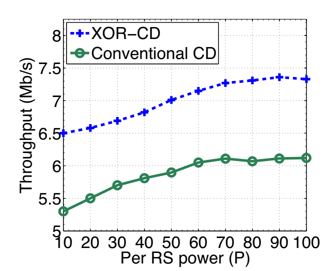


Fig. 8. Effects of power constraints for a 100m cell with 100 data subchannels, 30 relay subchannels, 10 MS, 3 RS.

For XOR-CD, it is preferable to encode the uplink and downlink of an MS with similar channel conditions, and assign the same relay subchannel to them. In case the channel conditions of uplink and downlink differ significantly, the link with worse channel condition will undermine the shared multicast rate. In Fig. 7 we explore the influence of path loss on the average throughput improvement over time. Path loss diversity across MS is increased when we increase the cell radius, since MS's differences in distances to BS are increased. For XOR schemes that encode packets from different links [4], the network coding gain diminishes and researchers proposed methods such as superposition codes to deal with this issue [10]. In contrast, XOR-CD takes advantage of bi-directional traffic on the *same* MS instead of encoding across two different MS. Though channel conditions of different MS differ severely as cell radius increases, uplink and downlink conditions of the same MS become more similar, since path loss becomes dominant over random fading. This explains the increased

TABLE II
THROUGHPUT VALUES OF DIFFERENT RELAY POWER PROFILES.

Relay power profile: RS1	RS2	RS3	Throughput (Mbps)	Improvement (%)
$10P$	$10P$	$10P$	6.69	—
$15P$	$9P$	$6P$	6.93	3.58
$15P$	$6P$	$9P$	6.91	3.29
$18P$	$7P$	$5P$	7.14	6.73
$18P$	$5P$	$7P$	7.11	6.28

throughput improvement of XOR-CD when cell radius is increased. Therefore, XOR-CD remedies the shortcoming of coding to the worse rate, and achieves better throughput improvement in large path loss case, which is substantially different from previous approaches of using XOR.

E. Performance of Power Allocation

Finally, we evaluate performance of XOR-CD when relay power is constrained. We implement the subgradient based power allocation algorithm *ALG3*, as well as its counterpart for the NO-XOR problem. We enforce a uniform power constraint for each of the RS. The per RS power constraint is such that each RS has relay power NP, where P is the power used for one direct transmission as in Sec. IV-B1. We vary N from 10 to 100 and obtain Fig. 8. We observe that XOR-CD is *less* sensitive to power constraints as reflected by the marginal improvement compared to conventional CD. This is because XOR-CD utilizes power more efficiently, resulting in a lower power demand at RS. Therefore, pumping more power does not improve its performance substantially.

We also study non-uniform power constraints at RS. For the same configuration and network topology, we fix the total power constraint to be $30P$ and vary different RS's individual power constraint. Table II summarizes the results of different relay power profiles. We observe that allocating more power into RS1 has a positive effect on the average throughput, while adjusting the constraints at RS2 and RS3 has little impact. The reason is that in our simulation RS are randomly located inside the cell. RS1 is located closest to the BS, providing a much better relay channel for cooperative transmissions. Allocating more power into RS1 boosts its relaying capacity and improves throughput. The result also suggests that power allocation at RS needs to be location-adaptive to best utilize resources.

VII. CONCLUSION AND FUTURE WORK

To the best of our knowledge, this work represents the first attempt to study XOR-assisted cooperative diversity in multi-channel networks. We seek to unravel two questions that have not yet been explored: first, how to effectively exploit network coding aided cooperative diversity when overhearing cannot be opportunistically harvested; second, how to reap different forms of gains available in such networks when relay assignment, relay strategy selection, channel assignment and power allocation intricately interplay with each other.

As our first contribution, we design XOR-CD, a novel cooperative diversity scheme with XOR in multi-channel networks. It capitalizes bi-directional traffic and is shown to be able to greatly improve relay efficiency and throughput by both information theoretical analysis and realistic simulations. As our second contribution, we propose a unifying optimization framework to exploit multi-user diversity, cooperative diversity and network coding jointly. We establish the NP-hardness of the problem, and propose efficient approximation algorithms with provably the *best* performance guarantee. Extensive simulation corroborates the effectiveness of our algorithms. In the future we plan to investigate XOR-CD in multi-hop multi-channel networks, where *scheduling* and *spatial reuse* pose new challenges to unleash its full potential.

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