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A Secondary Market for Spectrum

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***Abstract*—Dynamic spectrum trading amongst small cognitive users is fundamentally different along two axes: temporal variation, and spatial variation of user demand and channel condition. We advocate that a spectrum *secondary market*, analogous to the stock market, is to be established for users to dynamically trade among themselves their channel holdings obtained in the primary market from legacy owners. We design a market mechanism based on dynamic double auctions, creating a marketplace in the air to match bandwidth demand with supply. In the analysis we prove important economic properties of the mechanism, notably its truthfulness and asymptotic efficiency in maximizing spectrum utilization. Complimentary simulation studies corroborate that spectrum utilization and user performance can be improved by establishing the spectrum secondary market.**

I. Introduction

To address the discrepancy between the unabated growth of wireless bandwidth demand and limited spectrum resources, substantial efforts have been undertaken to redistribute the spectrum so that users in need can gain access and existing ones can obtain benefits by leasing their abundant spectrum [1]–[5]. Auctions [4], [5] are perceived to be fair and efficient candidate solutions to achieve such redistribution.

Conventional spectrum auctions, in general, are proposed under a *primary market* paradigm. Specifically, these auctions are performed weekly or daily with legacy spectrum owners on the selling side and cognitive users on the buying side. Channels are often modeled to be homogeneous, buyers are assumed to be static and have fixed demand, and interference information such as conflict graphs is globally available. From an economics perspective, such an approach parallels a primary market of the capital markets, and is suitable only to deal with issuance of relatively long-term spectrum leases from legacy owners to large cognitive users.

The key difference between our work and previous approach is that, we mainly focus on dynamic spectrum trading among small cognitive users themselves, *i.e.* the case where cognitive users are both the sellers and the buyers. By shifting from a "macro" to a more "micro" perspective, we observe that the underlying assumptions of the primary market paradigm no longer hold. For small users, traffic demand is extremely bursty as widely observed by many existing works [6]. Moreover, channel bandwidth is of a finer granularity, exhibiting significant time and frequency selectivity due to fading and user mobility as reported by extensive measurements [7]. The monolithic primary market paradigm designed for long-term

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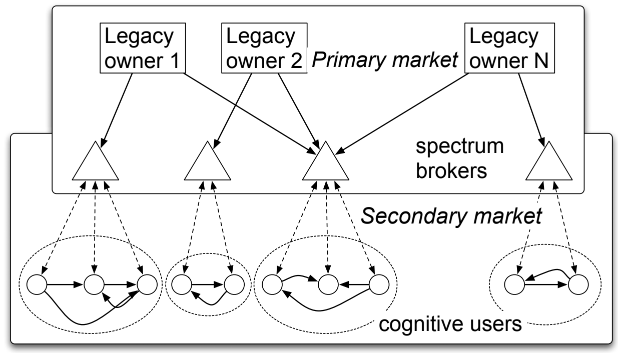

Fig. 1. The conceptual spectrum market structure for cognitive users. Legacy owners lease unused spectrum to spectrum brokers, each of which represents the aggregated demand of users of a certain area. The leased channels are then traded among cognitive users themselves dynamically in secondary market, with their respective spectrum broker as the auctioneer.

spectrum redistribution becomes inherently inefficient, if not detrimental, when applied to this scenario.

We advocate that a spectrum *secondary market* is to be established, as shown in Fig. 1. The secondary market works in harmony with primary market through "spectrum brokers." The leased spectrum resources from the primary market are traded dynamically amongst cognitive users themselves through the secondary market in a fine time scale (e.g. several minutes), to adapt to time-varying demand and channel condition. A user can sell some channels to others when traffic demand is reduced, or when the channels are in deep fading. The windfall from the sales can be used for future purchases when demand increases, or be exchanged for channels with better condition. By establishing the secondary market among cognitive users, the spectrum becomes more liquid and easier to obtain and relinquish, leading to more efficient utilization as implied by fundamental principles of economics [8].

Our contribution in this paper is a novel multi-unit *double auction* mechanism. In our double auction, each of the different channels is analogous to different stocks in the stock market, with dynamic prices to incorporate multi-channel and multi-user diversity. Through trading, channels are dynamically redistributed to maximize resource utilization. The mechanism is proven to be *truthful*, so users cannot expect a higher utility gain by cheating. Therefore, the dominant strategy is to report the true valuations in the bids or asks. It is also *asymptotically efficient*, in the sense that it maximizes the total utility gains obtained by all participating users asymptotically. To our knowledge this is the first auction tailored for the spectrum secondary market that guarantees both properties.

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II. The Spectrum Secondary Market Institution

We start by introducing the underlying network model as well as the basics of double auctions, and then the main design and characterization of the spectrum double auction, namely the *truthfulness* and *asymptotic efficiency*.

A. Network Model

We consider a micro-level cognitive radio network covered by one spectrum broker. We assume that channel reuse is possible in a macro level across micro networks represented by many spectrum brokers, and is taken care of by spectrum auctions in the primary market [4], [5]. For our micro network, OFDMA is used as the multi-access technology as recommended by the IEEE 802.22 draft [9]. It ensures that every user has a set of orthogonal channels, each of which is divided into a number of orthogonal subcarriers with identical bandwidth. We model channels by frequency-selective fading, with coherence bandwidth in the order of the bandwidth of a few channels. This implies that fading between channels far away from each other is uncorrelated, and each subcarrier of the same channel has the same fading statistics.

B. Basics of Double Auction

Intrinsically, the market mechanism among cognitive users is a double auction with multiple divisible *commodities*, as channels are heterogeneous across users. For each channel, it is a single-commodity double auction, and the subcarriers are the smallest trading units. Without loss of generality, we focus on an arbitrary round of a single-commodity double auction for an arbitrary channel in the subsequent analysis.

We introduce some definitions and notations. The *reservation price* is defined as the *true* highest price p_i^b per unit a buyer i is willing to offer, and the lowest price p_j^a a seller j is willing to accept. Its value is equal to each user's private valuation of the channel, and is unknown to other users and the spectrum broker, serving as the *auctioneer* here. Bid (ask) price b_i (a_j) is the *reported* highest (lowest) price buyer (seller) i (j) is willing to trade. Along with b_i (a_j) a buyer (seller) also submits q_i^b (q_j^a) indicating the maximum volume she intends to trade. $\hat{p}^b$ and $\hat{p}^a$ are the transaction prices at which winning buyers and sellers trade. The utility gain for a winning buyer is $u_i^b(b_i) = (p_i^b - \hat{p}^b)\hat{q}_i^b$, and for a winning seller is $u_j^a(a_j) = (\hat{p}^a - p_j^a)\hat{q}_j^a$, where $\hat{q}_i^b$ and $\hat{q}_j^a$ denote the trading volume.

C. Design of the Spectrum Double Auction

We illustrate in details the working of the spectrum double auction in this section. It consists of the following two phases.

*1) **Winner Determination**:* The spectrum broker sorts all orders so that

$$b_1 > b_2 > \cdots > b_n \tag{1}$$

and

$$a_1 < a_2 < \cdots < a_m. \tag{2}$$

where m and n denote the number of bids and asks in this round, respectively. Strict ordering relations are assumed, since if two buyers/sellers report the same price their volumes can be combined to form an equivalent bid/ask. The demand volumes are arranged according to the descending price order as shown in (1), and the supply volumes according to the ascending price order (2). There exists a critical point q^* where there are K bids and L asks such that:

$$a_{L+1} \geq b_K \geq a_L, \text{ and } \sum_1^{K-1} q_i^b \leq \sum_1^{L} q_j^a \leq \sum_1^{K} q_i^b, \text{ or} \tag{3}$$

$$b_K \geq a_L \geq b_{K+1}, \text{ and } \sum_1^{L-1} q_j^a \leq \sum_1^{K} q_i^b \leq \sum_1^{L} q_j^a \tag{4}$$

The first case — corresponding to Eq. (3) — is shown in Fig. 2. If Eq. (3) holds, $q^* = \sum_1^L q_j^a$; in case Eq. (4) holds, $q^* = \sum_1^K q_i^b$. If such a point cannot be found, *i.e.* the supply and demand curves do not intersect, q^* is simply the minimum of total supply $\sum_1^n q_j^a$ and demand $\sum_1^m q_i^b$.

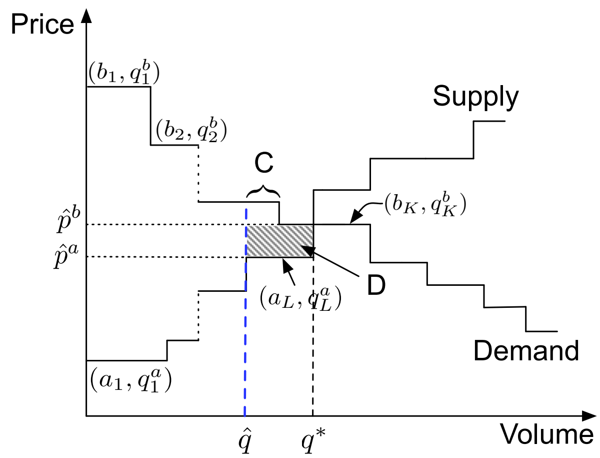

Fig. 2. Spectrum double auction design.

The total transaction volume $\hat{q}$ is then set as follows:

$$\hat{q} = \min\left(\sum_1^{L-1} q_j^a, \sum_1^{K-1} q_i^b\right).$$

Each bid with index less than K and each ask with index less than L will be involved in a trade, and are thus the winners of the auction.

*2) **Payment Determination**:* First we set the transaction price $\hat{p}^b$ per subcarrier to be b_K for winning buyers, and $\hat{p}^a$ to be a_L for winning sellers. Since $b_K < b_i, \forall i < K$ and $a_L > a_j, \forall j < L$, trading is profitable for both sides. Then we need to determine the trading volume for each winner. Note that because our mechanism supports multi-unit trading, we need to match the supply and demand volumes exactly. Hence it is possible that some orders can only be *partially* satisfied. Specifically, when $\sum_1^{K-1} q_i^b > \sum_1^{L-1} q_j^a$, as the case in Fig. 2, every ask with index less than L is satisfied. The bid b_{K-1}, however, cannot be fully executed. Instead of letting buyers corresponding to bid b_{K-1} suffer the shortage, we dictate that $\sum_1^{K-1} q_i^b - \sum_1^{L-1} q_j^a$ winning buyers are randomly chosen to sacrifice *one* subcarrier each to absorb the shortage. The purpose of doing so is to ensure bid-independence as we will show in Sec. II-D.

Similarly, when $\sum_1^{K-1} q_i^b < \sum_1^{L-1} q_j^a$, every bid with index less than K is satisfied, while a random set of $\sum_1^{L-1} q_j^a - \sum_1^{K-1} q_i^b$ sellers is chosen to sacrifice one subcarrier of supply each. Each winning buyer (seller) pays (receives) $\hat{p}^b$ ($\hat{p}^a$) times its total trading volume. The spectrum broker collects the trading surplus, $\hat{q}(\hat{p}^b - \hat{p}^a)$. This clears the market for a given channel. Note that we sacrifice the potential trading values from b_K and a_L, denoted by D, as well as the volume mismatch between supply and demand, C, in Fig. 2.

By the same mechanism, the spectrum broker clears the remaining orders for other channels. Transaction prices for each channel are also announced to every user. All outstanding orders are then removed. This concludes one round of trading.

D. Truthfulness

We present the proof of truthfulness of our double auction here. It consists of three steps: (1) We prove that the winner determination is *monotonic*. (2) We show that the payment determination algorithm is *bid-independent* for both buyers and sellers. (3) Based on these lemmas, we finally prove the truthfulness, *i.e.* no user can improve its utility by setting its bidding price other than the reservation price, by considering all possible outcomes of bidding truthfully and untruthfully.

*1) **Monotonic Winner Determination**:*

Lemma 2.1: If any buyer i wins by bidding b_i^1, it will also win if it bids b_i^2, where $b_i^2 > b_i^1$, provided all the other bids and asks remain the same.

Proof: We prove this lemma by contradiction. Consider two sorted lists of bids, B_1 and B_2. The bids of B_1 and B_2 are the same except in B_1 buyer i bids b_i^1, and in B_2 it bids b_i^2. Define the position of i in B_1 and B_2 as $pos(b_i^1)$ and $pos(b_i^2)$ respectively. Since $b_i^1 < b_i^2$, $pos(b_i^1) > pos(b_i^2)$. Since all the other bids and asks remain the same, the demand curves for B_1 and B_2 after $pos(b_i^1)$, and before $pos(b_i^2)$ are the same. Assume now that i loses by bidding b_i^2. Then the total trading volume $\hat{q}$ must be smaller than $\sum_{i=1}^{pos(b_i^2)} q_i^b$, and hence smaller than $\sum_{i=1}^{pos(b_i^1)} q_i^b$, which means i also loses by bidding b_i^1. This is a contradiction. Hence it cannot be that i wins in B_1 and not win in B_2. ■

Similarly we can prove the following lemma for sellers.

Lemma 2.2: If any seller j wins by asking a_j^1, it will also win if it asks a_j^2, where $a_j^2 < a_j^1$, provided all the other bids and asks remain the same.

*2) **Bid-independent Payment Determination**:*

Lemma 2.3: If buyer i wins by bidding b_i^1 and b_i^2, the expected total payment charged to i is the same, provided all the other bids and asks remain the same. If seller j wins by asking a_j^1 and a_j^2, the expected total payment received by j is the same, provided all the other orders remain the same.

Proof: We prove the case for the buyer. The case for the seller can be proved in a similar spirit. Without loss of generality, assume that $b_i^1 < b_i^2$. As in the proof of *Lemma* 2.1, we have $pos(b_i^1) > pos(b_i^2)$, and the demand curves for B_1 and B_2 after $pos(b_i^1)$ are the same. Since the supply curves are the same, they intersect at the same point with the demand curves for B_1 and B_2, and have the same set of winning buyers and sellers. Therefore the transaction price $\hat{p}^b$ is the same for B_1 and B_2, which is independent of i's bid. The bid volume is also the same for B_1, B_2. Further, i has the same probability to be chosen to sacrifice one unit of demand surplus, if any, since the total number of winning buyers does not change. Hence, by our payment determination algorithm, the expected payment charged to i is the same. ■

*3) **Truthfulness**:* Based on the above lemmas, we can prove the truthfulness of the spectrum double auction.

Theorem 2.4: The spectrum double auction mechanism is truthful with respect to the reservation price.

Proof: Suppose a buyer i with reservation price p_i^b reports $b_i \neq p_i^b$. Consider the outcomes of bidding p_i^b and b_i. There are four possible scenarios. (1) i loses for both cases. Then i has zero utility gain in both cases. (2) i wins by bidding p_i^b but loses by bidding b_i. This happens only if $b_i < p_i^b$ by *Lemma* 2.1. Then its utility gain is clearly non-zero and is no less than that when it bids truthfully (zero). Our claim holds. (3) i wins by bidding b_i but loses by bidding p_i^b. This happens only if $b_i > p_i^b$ by *Lemma* 2.1. In this case, let $\hat{p}^b(b_i)$ and $\hat{p}^b(p_i^b)$ be the transaction prices when i wins and loses. Immediately we have $\hat{p}^b(p_i^b) > p_i^b$. For i to win by bidding b_i, b_i must be at least larger than $\hat{p}^b(p_i^b)$, because if $b_i = \hat{p}^b(p_i^b)$, i still loses. It is easy to show that $\hat{p}^b(b_i) \geq \hat{p}^b(p_i^b)$ must hold since all other bids and asks remain the same. Therefore $\hat{p}^b(b_i) \geq \hat{p}^b(p_i^b) > p_i^b$, and hence i has negative utility gain when it bids b_i, which is no more than when it bids truthfully (zero). Our claim holds. (4) i wins in both cases. By *Lemma* 2.3, i is charged the same payment, leading to same utility gain.

From the above we can see that no buyer can obtain higher utility gain by bidding $b_i \neq p_i^b$. In a similar spirit we can show the same for all sellers. Then we conclude that no user has an incentive to bid untruthfully. ■

The essential reason for our mechanism being truthful is that every winner always buys or sells at the prices proposed by someone else, since simply letting each winner buy or sell at its own price will violate bid-independent payment determination, and cannot guarantee truthfulness [10]. Moreover, we dictate that among the winners, a random set of them are selected to take on the supply (demand) shortage, so that they cannot mitigate their trading volume loss by cheating their prices.

E. Asymptotic Economic Efficiency

An *efficient* market maximizes the total social welfare. We prove that, our periodic double auction is *asymptotically* efficient when the number of users is large, as it is impossible to strictly achieve both truthfulness and efficiency simultaneously [10]. We first prove a lemma to bound the expected difference between two consecutive bids/asks in the sorted lists. It is later used to bound the efficiency loss factor and prove the asymptotic result.

Lemma 2.5: Assume that the reservation prices of n bids are i.i.d. random variables with continuous density function $f(\cdot)$ defined on the interval $[\underline{b}, \overline{b}]$, and reservation prices of m asks

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are also i.i.d. random variables with PDF $g(\cdot)$ on $[\underline{a}, \overline{a}]$. Denote the minimum and maximum of f and g as follows:

$$\phi = \min f(x) > 0, \gamma = \min g(x) > 0,$$
$$\psi = \max f(x) > 0, \delta = \max g(x) > 0.$$

Assume that ϕ, γ, ψ and δ are bounded away from zero. Then we have the following result:

$$\frac{1}{\psi(m+1)} \leq E\big[b_i - b_{i+1}\big] \leq \frac{1}{\phi(m+1)},$$
$$\frac{1}{\delta(n+1)} \leq E\big[a_{i+1} - a_i\big] \leq \frac{1}{\gamma(n+1)}.$$

Proof: Refer to our technical report [11]. ■

Theorem 2.6: Our periodic double auction mechanism achieves 100% economic efficiency when the number of users scales to infinity. It is therefore asymptotically efficient.

Proof: We prove for the case when Eq.(3) holds. When $\sum_1^{K-1} q_i^b > \sum_1^{L-1} q_j^a$, as shown in Fig. 2, the social welfare loss can be represented by the sacrificed trading volumes from buyer K and seller L, denoted as D and the volume of supply shortage C. We have

$$D \leq (b_K - a_L) q_L^a \leq (a_{L+1} - a_L) q_L^a$$

and the value of C is bounded by

$$C \leq (b_1 - b_K) q_L^a.$$

Let $\Delta(K, L)$ denote the social welfare loss of our mechanism. Then by *Lemma* 2.5 we have the following:

$$E[\Delta(K, L)] = E[D + C] \leq \left(\frac{1}{\gamma(n+1)} + \frac{K-1}{\phi(m+1)}\right) E[q^a].$$

The maximum social welfare achievable, denoted as $\Theta(K, L)$, can be expressed as follows:

$$\Theta(K, L) = \sum_1^K q_i^b (b_i - b_K) + \sum_1^L q_j^a (a_L - a_j) + (b_K - a_L) \sum_1^L q_j^a.$$

Using *Lemma* 2.5, assuming q_i^b and q_j^a are i.i.d random samples equal in distribution to random variables q^b and q^a respectively, we can bound its expectation as

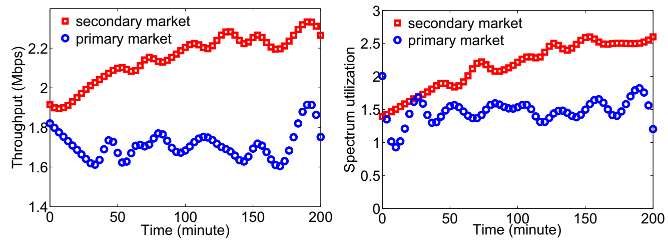
$$E[\Theta(K, L)] > \sum_1^{K-1} \frac{i \cdot E[q^b]}{\phi(m+1)} + \sum_1^{L-1} \frac{j \cdot E[q^a]}{\gamma(n+1)}.$$

We then can bound the efficiency loss factor $\rho(K, L)$:

$$\begin{aligned} \rho(K, L) &= E\left[\frac{\Delta(K, L)}{\Theta(K, L)}\right] \\ &< E[q^a] \left(\frac{1}{\gamma(n+1)} + \frac{K-1}{\phi(m+1)}\right) \frac{2\psi(m+1)}{K(K-1)E[q^b]} \\ &< \frac{E[q^a]}{E[q^b]} \left(\frac{2\psi(m+1)}{K(K-1)\gamma(n+1)} + \frac{2\psi}{K\phi}\right). \end{aligned}$$

Therefore, if K, the number of winning buyers, is large, the market inefficiency converges to zero at a rate no slower than $1/K^2$. In other words,

$$\rho(K, L) = \mathrm{O}\left(1/K^2\right).$$



(a) Throughput (b) Spectrum Utilization

Fig. 3. Performance of secondary market and primary market.

For the case when $\sum_1^{K-1} q_i^b \leq \sum_1^{L-1} q_j^a$ holds, we can prove that $\rho(K, L) = \mathrm{O}\left(1/L^2\right)$. Same conclusions can be proved when Eq. (4) holds. Hence the theorem. ■

Besides truthfulness and efficiency results, we also prove other economical properties of the auction mechanism for completeness, namely *ex-post individual rationality* and *budget balance*. Moreover, we prove the following:

Theorem 2.7: The spectrum double auction in each round runs in $O(n \log n + m \log m)$, where n and m is the number of bids and asks for a particular channel respectively.

Therefore it is highly efficient for implementation. Readers are directed to our technical report [11] for more details.

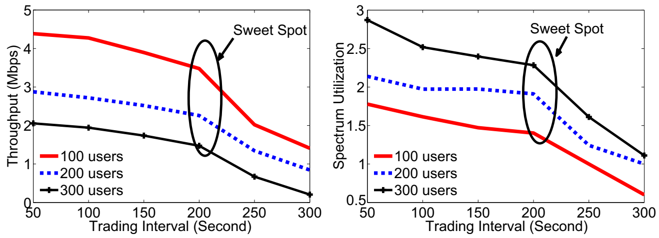
III. Simulation Results

We are now ready to resort to extensive simulations to study the performance of our spectrum secondary market design. As no previous work has been done for the secondary market, we rely on the double auction in [5] to serve as our performance benchmark, which represents state-of-the-art spectrum allocation in the primary market paradigm.

A. Simulation Settings

We use practical settings of an OFDMA cognitive radio network, including channel frequency, bandwidth, and adaptive modulation and coding schemes, as specified in the IEEE 802.22 draft [9]. There are a total of 48 channels, each of which contains 128 orthogonal subcarriers. Channel gain can be decomposed into a large-scale log normal shadowing with standard deviation of 5.8 and path loss exponent of 4 and a small-scale Rayleigh fading component. The inherent frequency selectivity is characterized by an exponential power delay profile with a delay spread 1.257μs. The time selectivity is captured by the Doppler spread, which depends on the user's speed. We assume every user moves around the network area according to the random waypoint model with its speeds (in km/h) following a uniform distribution $\mathrm{U}[0, 10]$. The combined complex gain is generated using an improved Jakes-like method [12].

We assume that data packets arrive at users following an asymptotically self-similar model, the ARIMA process, to model the bursty traffic [6]. All packets have the same size. The buffer is assumed to be sufficiently large, and the amount of data in it reflects user's demand. Two metrics are used to evaluate performance: *(1)* average user throughput, *(2)* spectrum utilization as the average utility from all users. We have also considered user demand satisfaction, but since it exhibits



(a) Throughput (b) Spectrum Utilization

Fig. 4. Investigation of the sweet spot of the market.

similar performance trend as that of spectrum utilization [11], we do not present the results here.

B. Overall Performance of the Secondary Market

We first evaluate the effectiveness of our spectrum secondary market. The simulation is performed for 200 minutes with carefully selected parameters and trading interval which we will explain in detail in later sections.

Fig. 3 shows the results. We observe that secondary market outperforms primary market based approach significantly for spectrum allocation among cognitive users. A 30% throughput gain and a 35% spectrum utilization gain are achieved on average, demonstrating the benefits provided by sensible secondary market design. The results clearly indicate that with the secondary market, every channel is traded as a different stock with dynamic prices across users, and is more efficiently utilized as time goes by, despite the temporal and spatial variation of user demand and link qualities.

C. Tradeoff Between Performance and Trading Overhead

As observed, the secondary market helps to significantly improve the performance. However, it is by no means a "free lunch," as there is communication overhead to transmit the orders and trading results among the users and the spectrum broker. Thus, we wish to obtain the optimal tradeoff between trading frequency and performance.

To address this question, we vary the length of the trading period in the spectrum market, and Fig. 4 shows the results. We observe that there is indeed a sweet spot. No matter what the market size is, we have a preferable tradeoff between the overhead and performance if trading is conducted around every 200 seconds. On one hand, performance decreases dramatically with longer trading intervals. On the other hand, more frequent trading substantially increases the overhead, with only marginal improvements. Thus, we may wish to set the market to operate every three minutes.

IV. Related Work

Existing works invariantly adopted a primary market based approach with primary users as sellers and secondary users as buyers as we discussed in Sec. I. Auction-based mechanisms have been studied extensively as solutions. In [1], spectrum band auctions have been proposed, where bidders obtain different spectrum channels to minimize interference. A truthful auction design is proposed in [4] based on the classical VCG auction. The revenue maximization of truthful spectrum auction is studied in [13]. Zhou *et al.* [5] have proposed to use double auctions for spectrum allocations recently. Their mechanism is designed for spectrum primary markets with homogeneous channels and fixed identical user demand. In contrast to all previous papers above, we explore the secondary trading scenario where the cognitive users are both sellers and sellers, *i.e.* trading happens amongst cognitive users themselves. Our mechanism adopts more realistic assumptions in the sense that it supports heterogeneous channels and multi-unit trading. To our knowledge, little prior work has been concerned about this trading scenario so far.

V. Concluding Remarks

We moved beyond the state-of-the-art that considers a primary market paradigm to establishing a secondary market for spectrum trading among cognitive users. In this work, we have presented a novel dynamic double auction mechanism, which makes it possible for users to bilaterally trade their channel holdings. The double auction is provably truthful and asymptotically efficient, and solves the dual challenges of temporal and spatial variation of traffic demand and channel condition. Though implementation calls for future research, we believe our design and characterization of the spectrum secondary market furthers the understand of and sheds light on efficient spectrum allocation among cognitive users.

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