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Efficient Resource Allocation with Flexible Channel Cooperation in OFDMA Cognitive Radio Networks

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***Abstract*—Recently, a cooperative paradigm for single-channel cognitive radio networks has been advocated, where primary users can leverage secondary users to relay their traffic. However, it is not clear how such cooperation can be exploited in multi-channel networks effectively. Conventional cooperation entails that data on one channel has to be relayed on exactly the same channel, which is inefficient in multi-channel networks with channel and user diversity. Moreover, the selfishness of users complicates the critical resource allocation problem, as both parties target at maximizing their own utility. This work represents the first attempt to address these challenges. We propose FLEC, a novel design of flexible channel cooperation. It allows secondary users to freely *optimize* the use of channels for transmitting primary data along with their own data, in order to maximize performance. Further, we formulate a unifying optimization framework based on Nash Bargaining Solutions to *fairly* and *efficiently* address resource allocation between primary and secondary networks, in both decentralized and centralized settings. We present an optimal distributed algorithm and sub-optimal centralized heuristics, and verify their effectiveness via realistic simulations.**

I. Introduction

Cognitive radio, with the ability to flexibly adapt its transmission parameters, has been considered as a revolutionary technology to dynamically access the under-utilized wireless spectrum [1]. Recently, a new paradigm in which primary users (PUs) can leverage secondary users (SUs) for their own transmissions, termed *cooperative cognitive radio networks* (CCRN), is advocated [2]. In CCRN, SUs cooperatively relay data for PUs in order to access the spectrum. A single channel network has been considered in [2]. The PU leases its channel to SUs for a fraction of time in exchange for cooperative transmission. SUs allocate a portion of the time fraction for relaying primary data, and the rest for their own traffic. Assuming they have better channel conditions to the primary receiver, cooperative relay can dramatically increase primary transmission rates. Meanwhile, SUs also gain opportunities to access the spectrum, resulting in a "win-win" situation.

In this paper, we investigate cooperative cognitive radio networks from a new perspective. We consider a multi-channel cellular network based on OFDMA, such as IEEE 802.16 [3] and 802.22 [4], with multiple SUs assisting PUs on the uplink as shown in Fig. 1. Multi-channel networks impose unique challenges of realizing the cooperative paradigm, as we narrate in the following, along with our original contributions to precisely address them.

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First, we observe that conventional user cooperation permeated through the cooperative diversity literature [5] becomes inefficient when directly applied to multi-channel CCRN. It implicitly postulates that data on one channel has to be relayed on *exactly* the same channel, which may not be amenable to relaying. Meanwhile, some other channel may have redundant relay capacity to incorporate additional data with little cost. In other words, cooperation using the same channel misses the bulk of PU-SU cooperation opportunities, by unnecessarily limiting the space of SU resource allocation to only the temporal dimension, as shown in Fig. 1.

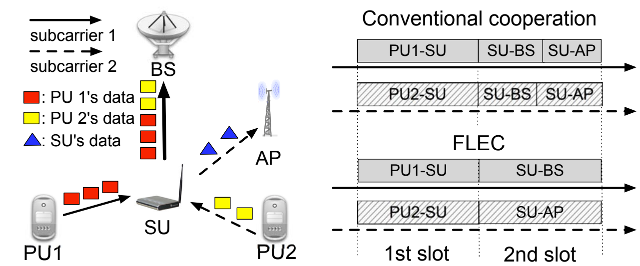

Fig. 1. The motivating scenario for *Flexible Channel Cooperation (FLEC)*.

Our *first* contribution in this paper is a new design for cooperation among SUs and PUs, termed Flexible Channel Cooperation (FLEC), that opens up all dimensions of resource allocation for SUs. It takes advantage of channel and user diversities profoundly available in multi-channel networks, and allows SU to freely *optimize* its use of resources — channels and time slots leased from PUs, as well as power — for relaying primary data along with its own data, as long as all the primary data it received can be delivered.

The basic idea of FLEC works as shown in Fig. 1. Time is equally divided into two slots among cooperating users. PUs transmit in the first slot to SU, and SU transmits in the second slot to the primary BS and to its own AP. SU strategically customizes its use of leased resources, *e.g.* it can use subcarrier 1 solely for relaying data aggregated from both subcarrier 1 and 2, and use subcarrier 2 solely for sending its own data as in Fig. 1. The intuition is that, if subcarrier 1 has superior conditions on SU-BS link but poor conditions on SU-AP link, it is much more efficient optimizing it to relay data from both subcarriers. Such channel *swapping* or *shuffling* results in boosted SU throughput, as well as larger relay capacity for PU, since the overall spectral efficiency is improved. Spectral efficiency gain can in turn be translated to more cooperation opportunities, as well as increased network capacity and performance.

The preceding description of FLEC assumes a decentralized setting where the primary and secondary networks are independent. Subcarriers are assigned to PUs by BS *a priori* to SU cooperation, and only those assigned to the helped PUs are leased to the respective helping SUs. In a centralized setting where SU cooperation becomes an integral part of the scheduling performed by the primary BS, it becomes possible to assign *any* subcarrier to a helping SU, to further improve the performance. We also consider the centralized FLEC in our paper. Note that in both settings, it is still possible to use one channel for *both* relay and SU's own data following an appropriate time sharing strategy, if the channel has excellent conditions on both SU-BS and SU-AP links. We will first assume this possibility, and later show that surprisingly, by restricting to *exclusive* channel usage, *i.e.* one channel is used for either relaying primary data or sending secondary data, the optimality of the solution is still preserved.

Given FLEC, the *second* unique challenge in multi-channel CCRN is how to schedule the transmissions and allocate resources, in order to maximize performance gains while ensuring fairness among all users. A SU may assist several PUs (as in Fig. 1) while a PU may also be paired up with several SUs, complicating the resource allocation problem. Moreover, in reality, PUs and SUs are selfish and have conflict of interests. If relaying is beneficial, PUs have a natural tendency to exploit SUs as much as possible by leasing more channels, while SUs striving to maximize their own throughput will no longer be willing to relay, given their depleted power budget. Additionally, PUs compete among themselves when one SU resides in a suitable position to relay for all of them; likewise SUs compete among themselves if one channel has good conditions on all of them. Our main objective in this paper, therefore, is to develop efficient yet fair resource allocation algorithms for FLEC in multi-channel networks.

To this end, our *second* contribution is a unifying optimization framework based on Nash Bargaining Game [6]. Our framework jointly considers relay and subcarrier assignment, relay strategy optimization, and power control. The solution found, called the Nash Bargaining Solution (NBS) [6], is a unique Nash Equilibrium point with NBS fairness among PUs and SUs, which is a generalized proportional fairness notion. We consider both decentralized and centralized FLEC as introduced above. In the decentralized case, we obtain an optimal distributed algorithm based on dual decomposition, to allow PUs to bargain with neighboring SUs autonomously. In the centralized case, cooperation opportunities are to be carefully *invented* and *engineered*, rather than distributively *harvested*. We identify the NP-hardness of the problem, design a three-step heuristic via a decoupling approach, and prove the approximation ratio for the decoupled subcarrier assignment algorithm. Both algorithms are highly efficient in that they can meet typical scheduling deadlines of 5–10 ms [3] in real OFDMA systems. Thus we believe our work sheds light on the design and implementation of OFDMA based cooperative cognitive radio networks.

II. An Optimization Framework

A. System Model

We start by introducing the system model. We consider the uplink of a single-cell OFDMA network. A number of SUs are located in the cell and perform cooperative transmission for PUs to access the primary spectrum. We assume that PUs and SUs have infinite backlogged data to send and the OFDM frames are synchronized. Decode-and-forward multi-hopping [5] is used when SUs relay primary data. Note that higher rates are achievable with more sophisticated coding/decoding schemes, *e.g.* maximum ratio combining at the destination based on the signals received in both slots (instead of multi-hopping) [5]. Here we focus on decode-and-forward multi-hopping only for simplicity of presentation. Our analysis and algorithms are readily applicable to scenarios with more complex coding/decoding schemes.

We model the fading environment by large scale path loss and shadowing, along with small scale frequency-selective Rayleigh fading. The coherence bandwidth is in the order of the width of a few subcarriers so that adjacent subcarriers have similar channel conditions. Fading between subcarriers in different frames is independent, and remains stable during each frame. We assume techniques for channel estimation are employed and full channel side-information (CSI) is available, which makes the optimization possible. Such assumptions about the fading environment and CSI are commonly used as in [7]–[10]. Noises are modeled as i.i.d. circularly symmetric complex Gaussian noises $\mathcal{CN}(0, N_0 W)$.

There are a total of K subcarriers, N_P primary users and N_S secondary users in the network. Let $N = N_P + N_S$. One subcarrier can be allocated to one PU only, and can be leased to one SU only. For a given PU $i \in [1, N_P]$, if subcarrier c with bandwidth W and complex channel gain h_i^c is allocated for direct transmission, the achievable throughput is:

$$R_{i,N+1}^c = W \log\left(1 + p_i^c \cdot g_i^c\right), \text{where } g_i^c = \frac{|h_i^c|^2}{\Gamma N_0 W}. \tag{1}$$

The subscript $(N+1)$ is used to denote the direction transmission mode. Γ is the coding gap to capacity and p_i^c denotes the allocated power. Without loss of generality W equals 1 in the subsequent analysis. If PU i decided to lease c to SU $j \in [N_P + 1, N]$ for cooperative transmission, then in the first time slot, the achievable throughput on PU-SU link is

$$R_{i,j}^c = \frac{1}{2} \log\left(1 + 2p_i^c g_{i,j}^c\right), \tag{2}$$

since the effective power and throughput should take into account the two-slot structure of cooperative transmission. For SU j in the second time slot, under FLEC, it can freely decide whether to use c solely for relay, or solely for its own data, or jointly for both purposes in a time sharing manner. W.L.O.G., let $\alpha_j^c \in [0, 1]$ denote its relay strategy. Then j's throughput for relay and its own transmission is as follows, respectively:

$$R_{j,P}^c = \frac{1 - \alpha_j^c}{2} \log\left(1 + 2p_j^c g_{j,P}^c\right), R_j^c = \frac{\alpha_j^c}{2} \log\left(1 + 2p_j^c g_j^c\right). \tag{3}$$

Note that with conventional protocols, $R^c_{i,j} = R^c_{j,P}$ holds for any c PU i leases to SU j. With FLEC, this does not have to hold for every leased subcarrier. The only requirement is that SU j should deliver all data from the cooperating PUs, *i.e.* a total flow conservation requirement as follows:

$$\sum_{c=1}^{K}\sum_{i=1}^{N_P} R^c_{i,j} \leq \sum_{c=1}^{K} R^c_{j,P}, \forall j \in [N_P+1, N]. \quad (4)$$

B. Basics of Nash Bargaining Solutions

We present the salient concepts and results from Nash Bargaining Solutions, which are used in the sequel. For details we refer readers to [6].

The basic setting is as follows: Let $\mathbf{N} = \{1, 2, \ldots, N\}$ be the set of players, including PUs and SUs. Let $\mathbf{S}$ be a closed and convex subset of $\mathcal{R}^N$ to represent the set of feasible payoff allocations that players can get if they all work together. Let $R^{\min}_n$ be the minimal payoff that the nth player would expect; otherwise, he will not cooperate. Suppose $\{R_n \in \mathbf{S} | R_n \geq R^{\min}_n, \forall n \in \mathbf{N}\}$ is a nonempty bounded set. Define $\mathbf{R}^{\min} = (R^{\min}_1, \ldots, R^{\min}_N)$, then the pair $(\mathbf{S}, \mathbf{R}^{\min})$ is called a N-person bargaining problem.

Within the feasible set $\mathbf{S}$, we first define the notion of Pareto optimality as a selection criterion in a typical game setting.

Definition 1: The point $(R_1, \ldots, R_N)$ is said to be **Pareto optimal** if and only if there is no other allocation R'_n such that $R'_n \geq R_n, \forall n$, and $R'_n > R_n, \exists n$, *i.e.* there exists no other allocation that leads to superior performance for some user without inferior performance for some other user.

The question that arises is: at which of (infinitely many) Pareto-optimal points should we operate the system? A possible further criterion is the fairness of resource sharing. In this paper, we use the NBS fairness axioms from game theory. The intuitive idea is that after the minimal requirements are met for all users, the rest of the resources are allocated *proportionally* to users according to their conditions.

Definition 2: $\bar{\mathbf{r}}$ is a **NBS**, *i.e.* $\bar{\mathbf{r}} = \phi(\mathbf{S}, \mathbf{R}^{\min})$, if the following axioms are satisfied: *Feasibility, Pareto Optimality, Independence of Irrelevant Alternatives, Independence of Linear Transformations*, and *Symmetry* [6].

Theorem 1: There is a unique solution function $\phi(\mathbf{S}, \mathbf{R}^{\min})$ that satisfies all axioms in *Definition 2* such that [6]

$$\phi(\mathbf{S}, \mathbf{R}^{\min}) \in \arg \max_{\mathbf{R} \in \mathbf{S}, R_n > R^{min}_n} \prod_{n=1}^{N} \left(R_n - R^{\min}_n\right). \quad (5)$$

C. An Optimization Framework Based on NBS

For our problem, we wish to consider *long-term* NBS fairness, which depends on the average throughput gain from cooperation. For elastic traffic, long-term fairness not only faithfully reflects users' perceived performance, but also gives more flexibility to exploit multi-user diversity. As discussed above, the cooperative game in an OFDMA based CCRN can be formulated as follows. Each user, being primary or secondary, has $\bar{R}_n$, the average throughput, as its objective function. It is bounded above and has a nonempty, closed, and convex support. The goal is to maximize all $\bar{R}_n$ simultaneously. $\bar{\mathbf{R}}^{\min}$ represents the minimal performance requirement. For PUs, the minimal requirement will be the optimal average throughput they could obtain should they choose not to cooperate with SUs, given by a multi-user uplink scheduling algorithm [11]. For SUs, their minimal requirement is simply zero. $\mathbf{S}$ is the feasible set of resource allocation with FLEC that satisfies $\bar{R}_n > \bar{R}^{\min}_n \ \forall n$. The problem, then, is to find the NBS, *i.e.* to solve the optimization problem (5) with $\bar{R}_n$ and $\bar{R}^{\min}_n$.

The product terms in (5) make the problem difficult to solve. We first show that mathematically, it is equivalent to solving the following:

$$\max_{\bar{\mathbf{R}} \in \mathbf{S}, \bar{R}_n > \bar{R}^{\min}_n} \sum_{n=1}^{N} \ln\left(\bar{R}_n - \bar{R}^{\min}_n\right). \quad (6)$$

This utility maximization problem has to be solved in every scheduling epoch as channel qualities change over time. It has been shown that maximizing the aggregate *marginal* utility $\sum U'(\bar{R}_n) \cdot R_n$ at each epoch achieves long-term utility maximization [12]. Therefore, separating the terms for PUs and SUs, the basic resource allocation framework for OFDMA cooperative cognitive radio networks at each epoch t is:

$$\max_{\mathbf{R} \in \mathbf{S}, \mathbf{R} \succ \mathbf{0}} \sum_{i=1}^{N_P} \frac{R_i - R^{\min}_i}{\bar{R}_i(t) - \bar{R}^{\min}_i(t)} + \sum_{j=N_P+1}^{N} \frac{R_i}{\bar{R}_i(t)}. \quad (7)$$

$R_i, \bar{R}_i(t), R_j, \bar{R}_j(t)$ denote the instantaneous and average throughput for PU i and SU j at epoch t, respectively. $R^{\min}_i, \bar{R}^{\min}_i(t)$ are the instantaneous and average throughput requirement, which can be obtained by running a multi-user scheduling algorithm at each epoch, such as [11], and using the weighted averaging technique.

III. Optimal Distributed Algorithm

A. Problem Formulation

We first consider a decentralized setting where the secondary network is independent from the primary network, and cannot be controlled by the primary BS. Thus, BS allocates resources to PUs *a priori* to any cooperative transmission, and SUs have to "negotiate" distributively with PUs in order to have cooperation taking place. In other words, cooperative transmission serves as an add-on component to the existing primary network, and is *opportunistically* harvested. This may correspond to the most immediate implementation scenario of CCRN that does not call for any change in the existing primary infrastructure, and therefore is of great practical interest.

Resource allocation problem in this setting, including relay assignment, SU subcarrier assignment, SU relay strategy optimization using FLEC, and PU-SU power control within the basic framework in Sec. II-C can be expressed succinctly as:

$$\text{Distributed:} \max_{\mathbf{R}, \mathbf{P} \succ \mathbf{0}, \boldsymbol{\alpha}} \sum_{i=1}^{N_P} \frac{R_i - R^{\min}_i}{\bar{R}_i(t) - \bar{R}^{\min}_i(t)} + \sum_{j=N_P+1}^{N} \frac{R_j}{\bar{R}_j(t)}$$
$$\text{s.t. } \mathbf{P} \cdot \mathbf{1}^T \preceq \mathbf{p}^{\max}, \mathbf{R} \in \mathcal{C}(\mathbf{P}, \boldsymbol{\alpha}), \quad (8)$$

where $\mathbf{p}^{\max} = [p_1^{\max}, \ldots, p_N^{\max}]^T$ is the power constraint vector, $\mathbf{P}$ is an $N \times K$ matrix such that P_n^c denotes the power expended by user n in subcarrier c, $\boldsymbol{\alpha}$ is an $N_S \times K$ matrix such that α_j^c denotes the FLEC strategy of SU j on c, and $\mathcal{C}(\cdot)$ denotes the achievable rate region given $\mathbf{P}$ and α (Eq. (1)–(3)), with the flow conservation constraint at each SU (Eq. (4)). Since only one PU and one SU can be active on each subcarrier, the column vector $\mathbf{P^c}$ has at most two non-zero entries, and it also specifies relay and subcarrier assignments.

B. Dual Decomposition

The decentralized problem (8) is essentially a mixed integer program, with the objective function being neither convex nor concave. However, in an OFDMA system with many narrow subcarriers, the optimal solution is always a convex function of $\mathbf{p}^{\max}$, because if two sets of throughputs using two different sets of $\mathbf{P}$ and $\boldsymbol{\alpha}$ are achievable individually, their linear combination is also achievable by a frequency-division multiplexing of the two sets of strategies. In particular, using the duality theory of [13], the following is true:

Proposition 1: The decentralized resource allocation problem (8) has zero duality gap in the limit as the number of OFDM subcarriers goes to infinity, even though the discrete selection of subcarriers, SUs and relay strategies are involved.

This proposition allows us to solve non-convex problems in their dual domain. Note that although the proposition requires the number of subcarriers to go to infinity, in reality the duality gap is very close to zero as long as the number of subcarriers is large [9].

Introduce Lagrangian multiplier vectors $\boldsymbol{\lambda}, \boldsymbol{\mu}$ to the power and flow conservation constraints and the Lagrangian becomes

$$L(\mathbf{R}, \mathbf{P}, \boldsymbol{\alpha}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{N_P} \frac{R_i - R_i^{\min}}{\bar{R}_i(t) - \bar{R}_i^{\min}(t)} + \sum_{j=N_P+1}^{N} \frac{R_j}{\bar{R}_j(t)} + \sum_{n=1}^{N} \lambda_i \left(p_n^{\max} - \sum_{c=1}^{K} p_n^c \right) + \sum_{j=N_P+1}^{N} \mu_j \left(\sum_{c=1}^{K} R_{j,P}^c - \sum_{c=1}^{K} \sum_{i=1}^{N_P} R_{i,j}^c \right) \quad (9)$$

The dual function becomes

$$g(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \begin{cases} \max\limits_{\mathbf{R}, \mathbf{P}, \boldsymbol{\alpha}} & L(\mathbf{R}, \mathbf{P}, \boldsymbol{\alpha}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{s.t.} & \mathbf{R} \in \mathcal{C}(\mathbf{P}, \boldsymbol{\alpha}). \end{cases} \quad (10)$$

To solve $g(\boldsymbol{\lambda}, \boldsymbol{\mu})$ with given $\boldsymbol{\lambda}, \boldsymbol{\mu}$, it is equivalent to solving the problem with the following objective:

$$\sum_{c=1}^{K} \left(\sum_{i=1}^{N_P} \sum_{j=N_P+1}^{N+1} \frac{R_{i,j}^c}{\bar{R}_i(t) - \bar{R}_i^{\min}(t)} + \sum_{j=N_P+1}^{N} \frac{R_j^c}{\bar{R}_j(t)} - \sum_{i=1}^{N_P} \lambda_i p_i^c - \sum_{j=N_P+1}^{N} \lambda_j p_j^c + \sum_{j=N_P+1}^{N} \sum_{i=1}^{N_P} \mu_j \left(R_{j,P}^c - R_{i,j}^c \right) \right)$$

Notice that in the first term of the objective, j could be $N+1$ which corresponds to the possibility of direct transmission.

Therefore, the problem can be decomposed into K per-subcarrier problems. Recall that each subcarrier is already assigned to a PU by the BS, the per-subcarrier problem then reduces to finding the optimal helping SU and the optimal resource allocation, and can be shown alternatively as follows:

$$\text{Per-subcarrier:} \max_{j, p_i^c, p_j^c, \alpha_j^c} \frac{R_{i,j}^c}{\bar{R}_i(t) - \bar{R}_i^{\min}(t)} + \frac{R_j^c}{\bar{R}_j(t)} - \lambda_i p_i^c - \lambda_j p_j^c + \mu_j \left(R_{j,P}^c - R_{i,j}^c \right)$$
$$\text{s.t. } R_{i,j}^c, R_j^c \in \mathcal{C}(p_i^c, p_j^c, \alpha_j^c). \quad (11)$$

C. Solving the Per-Subcarrier Problem

Next we show the per-subcarrier problem can be solved efficiently via an exhaustive search over all SUs and transmission strategies (direct or cooperative). To enable such exhaustive search, we need to derive solutions under direct or cooperative transmission modes for a given subcarrier c with its PU i.

$\triangleright$ *Direct Transmission*

If PU i chooses direct transmission, (11) becomes

$$\max_{p_i^c} \frac{\log(1 + p_i^c g_i^c)}{(\bar{R}_i(t) - \bar{R}_i^{\min}(t))} - \lambda_i p_i^c, \quad (12)$$

the solution of which is readily available by simple calculus:

$$\tilde{p}_i^c = \left[\frac{1}{\lambda_i \left(\bar{R}_i(t) - \bar{R}_i^{\min}(t) \right)} - \frac{1}{g_i^c} \right]^+. \quad (13)$$

$\triangleright$ *Cooperative Transmission*

Substituting the rate formulas (2)–(3) into (11) and regrouping the terms, the objective (11) becomes

$$\frac{\log(1 + 2p_i^c g_{i,j}^c)}{2(\bar{R}_i(t) - \bar{R}_i^{\min}(t))} - \frac{\mu_j \log(1 + 2p_i^c g_{i,j}^c)}{2} - \lambda_i p_i^c + \frac{\alpha_j^c \log(1 + 2p_j^c g_j^c)}{2\bar{R}_j(t)} + \frac{\mu_j (1 - \alpha_j^c) \log(1 + 2p_j^c g_{j,P}^c)}{2} - \lambda_j p_j^c$$

The first three terms, denoted as $b_i(j, \lambda_i, \mu_j)$, represent PU i's benefit by having SU j as its relay, discounted by possible violation of flow conservation with price μ_j and power expenditure with price λ_i. $b_i(j, \lambda_i, \mu_j)$ can be easily optimized by i as only p_i^c is involved:

$$\tilde{p}_i^c = \frac{1}{2} \left[\frac{1}{\lambda_i (\bar{R}_i(t) - \bar{R}_i^{\min}(t))} - \frac{\mu_j}{\lambda_i} - \frac{1}{g_{i,j}^c} \right]^+. \quad (14)$$

The last three terms, denoted as $b_j(i, \lambda_j, \mu_j)$, represent SU j's benefits from transmitting both its own and PU i's data, discounted by the power expenditure with price λ_i. Two optimizing variables α_j^c and p_j^c are involved here. Notice that the terms are essentially a convex combination with α_j^c, we have the following theorem establishing the optimality of exclusive channel usage on SUs.

Theorem 2: For the distributed resource allocation problem (8), the optimal solution can be found, where every subcarrier c leased to SU j is *exclusively* used for either relaying primary data or sending its own data.

Proof: It follows from applying the fact that the convex combination of two functions is upper bounded by the maximum of the two functions, *i.e.,*

$$af_1(x) + (1-a)f_2(x) \le \max(f_1(x), f_2(x)), a \in [0,1].$$

The per-subcarrier problem (11) can be solved optimally with $\alpha_j^c = \{0,1\}$, hence the dual and the primal problem (8). ■

Theorem 2 greatly simplifies the optimization problem as well as implementation since no complicated time sharing is needed to achieve optimal performance. Hence we let $\alpha_j^c = \{0,1\}$. Maximization of $b_j(i, \lambda_j, \mu_j)$ can be obtained by setting α_j^c to 0 and 1, deriving the optimal p_j^c respectively as shown in the following, and comparing the objective values. Ties can be broken arbitrarily.

$$\tilde{p}_j^c = \begin{cases} \frac{1}{2}\left[\frac{\mu_j}{\lambda_j} - \frac{1}{g_{i,P}^c}\right]^+, & \text{when } \tilde{\alpha}_j^c = 0, \\ \frac{1}{2}\left[\frac{1}{\lambda_j R_j(t)} - \frac{1}{g_i^c}\right]^+, & \text{when } \tilde{\alpha}_j^c = 1. \end{cases} \quad (15)$$

To summarize, the per-subcarrier problem (11) can be efficiently solved via exhaustive search over a finite set defined by the transmission strategies, SUs, and SU relay strategies with FLEC as discussed above. The size of this discrete set is very limited, making it feasible for a practical network.

D. A Distributed Algorithm

We have shown that the dual function can be decomposed into K per-subcarrier problems, the optimal solutions of which can be obtained efficiently through exhaustive search. Then, the primal problem (8) can be optimally solved by minimizing the dual objective:

$$\begin{aligned} \text{minimize} \quad & g(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{subject to} \quad & \boldsymbol{\lambda}, \boldsymbol{\mu} \succeq \mathbf{0}. \end{aligned} \quad (16)$$

Subgradient method can be used to solve this dual problem. The updating rules are as follows:

$$\lambda_n^{(l+1)} = \left[\lambda_n^{(l)} + \nu_n^{(l)}\left(\sum_{c=1}^{K} \tilde{p}_n^c - p_n^{\max}\right)\right]^+, \quad (17)$$

$$\mu_j^{(l+1)} = \left[\mu_j^{(l)} + \kappa_j^{(l)}\left(\sum_{c=1}^{K}\sum_{i=1}^{N_P} \tilde{R}_{i,j}^c - \sum_{c=1}^{K} \tilde{R}_{j,P}^c\right)\right]^+, \quad (18)$$

Following a diminishing step size rule for choosing $\boldsymbol{\nu}^{(l)}, \boldsymbol{\kappa}^{(l)}$, the subgradient method above is guaranteed to converge to the optimal dual variables. The optimal primal variables can then be easily found.

Observe that, because of the dual decomposition, dual optimization by a subgradient method can be done in a *distributed* fashion. The algorithm can be perceived as an iterative bargaining process. During each iteration, the per-subcarrier problems (11) can be solved simultaneously by the PU of the subcarrier exchanging information with neighboring SUs, though the objective jointly involves PU and SU's benefits. Specifically, from (14), the PU needs to know the current relay "price" μ_j from j in order to calculate the optimal power $\tilde{p}_i^c$, and the optimal value of its benefits $\tilde{b}_i(j, \lambda_i, \mu_j)$. The optimal $\tilde{\alpha}_j^c, \tilde{p}_j^c$ can be obtained by SU j only with its local information as in (15). Then the optimal value of SU's benefits $\tilde{b}_j(i, \lambda_j, \mu_j)$ needs to be passed to PU i. After calculating $\tilde{b}_i(j, \lambda_i, \mu_j)$ and collecting $\tilde{b}_j(i, \lambda_j, \mu_j)$ from all SUs, i can solve (11) by an exhaustive search to maximize $\tilde{b}_i(j, \lambda_i, \mu_j) + \tilde{b}_j(i, \lambda_j, \mu_j)$.

The subgradient updates can be easily performed by each and every primary and secondary users. While the dual variable λ_n is kept at each user privately and serves as a price signal to regulate its power consumption, μ_j is exchanged between PUs and SUs and serves as a relay price signal to coordinate the level of cooperation. When the relay traffic demand $\sum_{c=1}^{K}\sum_{i=1}^{N_P} \tilde{R}_{i,j}^c$ from PUs exceeds the supply $\sum_{c=1}^{K} \tilde{R}_{j,P}^c$ from j, *i.e.* PUs over-exploit j, j increases its relay price μ_j for the next round of bargaining to suppress the excessive demand, as shown in (18). Similarly, if j has redundant relay capacity $\sum_{c=1}^{K} \tilde{R}_{j,P}^c > \sum_{c=1}^{K}\sum_{i=1}^{N_P} \tilde{R}_{i,j}^c$, it will decrease the relay price μ_j to attract more relay traffic and therefore obtain more channels to use. The process continues until it converges to the optimal resource allocation.

Algorithm 1 *Distributed Bargaining*

1. Each primary user initializes the power price $\lambda_i^{(0)}$. Each secondary user initializes both power and relay prices $\lambda_j^{(0)}, \mu_j^{(0)}$.
2. Given $\boldsymbol{\lambda}^{(l)}, \boldsymbol{\mu}^{(l)}$, each PU i bargains with each neighboring SU j concurrently to solve the per-subcarrier resource allocation problem (11) using (13)-(15).
3. PU i performs a subgradient update for λ_i, and SU j performs a subgradient update for λ_j, μ_j as in (18) respectively.
4. Return to step 2 until convergence.

The complete bargaining algorithm is shown in **Algorithm 1**. We now analyze the amount of message exchanges and complexity here. For each pair of PU-SU, two messages $\mu_j, \tilde{b}_j(i, \lambda_j, \mu_j)$ need to be exchanged. They can easily be piggybacked in the probing packets from SU to PU to measure the channel gain, resulting in zero message exchange overhead. The complexity of solving K per-subcarrier problems by exhaustive search is $O(KN_S)$. The complexity of the subgradient update is polynomial in the dimension of the problem K. Therefore, the complete algorithm has complexity polynomial in KN_S. While this may render it infeasible for real-time scheduling within 5–10 ms when the network scales, the distributed nature of the algorithm makes it possible for each PU to *concurrently* solve the per-subcarrier problem, reducing the complexity to only $O(N_S)$. Further, in reality, only a few SUs residing in the neighborhood of the PU can potentially help and thus have to be considered. Therefore from the network point of view, each round of bargaining has complexity $O(1)$. Careful readers may be concerned with the slow convergence of the subgradient updates. We comment that since each PU only needs to bargain with neighboring SUs, the convergence complexity becomes polynomial in the limited size of the neighborhood. We observe

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in simulations — shown later in Sec. V-C — that the algorithm converges within about 20 iterations in most cases.

IV. Near-Optimal Centralized Algorithms

We now proceed to the centralized setting. We consider the scenario where the SU cooperative transmission becomes an integral part of primary BS scheduling, and SUs abide by the scheduling decisions, provided that the resource allocation is fair as reflected by the NBS fairness. With centralized FLEC, we have an additional dimension to optimize: *joint subcarrier assignment to PUs and SUs*.

While the problem can be formulated in a similar way as the decentralized problem (8) and solved via dual decomposition and subgradient update, it is computationally prohibitive to do so. At each iteration, the solution of the per-subcarrier problem now has to exhaustively search all possible combinations of PUs and their neighboring SUs, which has a complexity of $O(KN_P)$ since distributed concurrent optimization is not possible. Moreover, because of the global impact of centralized subcarrier assignment, the N-dimensional and N_S-dimensional dual variables $\boldsymbol{\lambda}, \boldsymbol{\mu}$ have to be numerically updated, the convergence of which is too slow to be useful for scheduling on a fast time scale as will be shown in Sec. V-C. Given that the simpler primary user scheduling without SU cooperation is extremely difficult in itself [11], we focus on developing efficient heuristics in this section, which reduce the complexity while exhibiting good performance. Nevertheless, the slow subgradient based centralized algorithm, called *Centralized Optimization* hereafter, is used to derive the optimal performance as a benchmark as in Sec. V-D.

A. Overview of the Heuristic Algorithm

To make the problem more tractable, we decouple it to three orthogonal dimensions: relay assignment, subcarrier assignment with FLEC, and power control. First we derive optimal relay assignment using bipartite matching, assuming that each SU is only able to help one distinct PU. This simplification is reasonable as it ensures a certain level of fairness. Then we assume that power is equally distributed, and derive subcarrier assignment algorithms. Even with optimal relay and equal power assignment, this turns out to be an NP-hard problem. We propose a sub-optimal algorithm based on randomized rounding and prove its approximation ratio. Finally, power allocation is solved to maximize performance with the given subcarrier assignment. Be reminded that as an initialization step, the BS first performs a multi-user scheduling [11] to determine $R_i^{\min}, \bar{R}_i^{\min}(t)$ for PUs before the three component algorithms run. The entire heuristic algorithm is called *Centralized Heuristic* hereafter. We note that other heuristics are possible, which are beyond the scope of this paper and left as our future research.

B. Sensible Relay Assignment

Here, we model each user n as having an *imaginary* channel with normalized channel gain to noise ratio $\bar{g}_n^c = \frac{1}{K}\sum_c g_n^c$ and power $p_n^{\max}$. Then the optimal FLEC strategy reduces to simple time-sharing on this channel. Assuming each SU can only help one distinct PU, the optimal relay assignment under the basic framework in Sec. II-C can be determined by:

$$\max_{x_{i,j}\in\{0,1\}} \sum_{i=1}^{N_P}\sum_{j=N_P+1}^{N+1} x_{i,j}\left(\frac{R_{i,j}-R_i^{\min}}{\bar{R}_i(t)-\bar{R}_i^{\min}(t)}+\frac{R_{j,i}}{\bar{R}_j(t)}\right)$$

$$\text{s.t. } R_{i,j}=\frac{1}{2}\min\left(\log(1+2p_i^{\max}\bar{g}_{i,j}),\log(1+2p_j^{\max}\bar{g}_{j,P})\right),$$

$$R_{j,i}=\frac{1}{2}\log(1+2p_j^{\max}\bar{g}_j)\left[1-\frac{\log(1+2p_i^{\max}\bar{g}_{i,j})}{\log(1+2p_j^{\max}\bar{g}_{j,P})}\right]^+,\quad \forall j\in[N_P+1,N],$$

$$R_{i,N+1}=\log(1+p_i^{\max}\bar{g}_i), R_{N+1,i}=0, \bar{R}_{N+1}(t)=\infty.$$

Here $x_{i,j}$ is the binary variable denoting the relay assignment of SU j to PU i. Note again that j can be $N+1$ to denote direct transmission.

The above relay assignment can be optimally solved by weighted bipartite matching. Construct a bipartite graph $A = (V_1 \times V_2, E)$ where V_1 and V_2 correspond to the set of PUs and SUs respectively. We patch void vertices to V_2 to make $|V_2| = |V_1| = N_P$, since the number of SUs is typically smaller. The edge set E corresponds to N_P^2 edges connecting all possible pairs of users in the two vertex sets. Each edge (i, j) carries a weight, $w_{i,j}$, where

$$w_{i,j}=\frac{R_{i,j}-R_i^{\min}}{\bar{R}_i(t)-\bar{R}_i^{\min}(t)}+\frac{R_{j,i}}{\bar{R}_j(t)}.$$

For edges connecting PUs to void SUs that we patched, the edge weights have captured the maximum marginal utility given by direct transmission. Observe that A is bipartite, optimal relay assignment is then equivalent to finding maximum weighted bipartite matching on A. The Hungarian algorithm is a popular polynomial-time algorithm to solve it optimally [14].

C. Subcarrier Assignment by Randomized Rounding

For PUs using direct transmission as determined by optimal relay assignment, they do not share resources with SUs, and as such cannot benefit from SU cooperation. Therefore they use the same subcarriers as allocated in the initialization step. For the set of PUs $\mathcal{N}_\mathcal{P}^\mathcal{R}$ that use cooperative transmission, the set of their allocated subcarriers $\mathcal{K}^\mathcal{R}$ in the initialization step will be collected and re-assigned by the following algorithm. For each PU i and its unique helping SU $j(i)$, we assume they will use power $\bar{p}_i = \frac{p_i^{\max}}{K_i}, \bar{p}_{j(i)} = \frac{p_{j(i)}^{\max}}{K_i}$ respectively on each subcarrier, where K_i is the number of subcarriers allocated to i in the initialization step [11]. Such an equal power assumption is widely used and leads to subcarrier assignment algorithms with near-optimal performance, as reported extensively [11], [15] and will be shown in Sec. V-D.

As seen in *Theorem 2*, assigning a subcarrier exclusively for relaying or transmitting SU's own data does not lose optimality in the decentralized case. This can also be proved similarly for the centralized case. Thus, the subcarrier assignment problem can be formulated as in (19), where $w_{i,j(i)}^{c_1}$ denotes the marginal utility (normalized to a large value $w_{\max}$) obtained by PU i on being assigned c_1 on PU-SU link in the first time slot (*i.e.* $w^{c_1}_{i,j(i)} = 0.5\log(1+2\bar{p}_i g^{c_1}_{i,j})/(\bar{R}_i(t) - \bar{R}^{\min}_i(t))$), and $w^{c_2}_{j(i),P}$ denotes the marginal utility of assigning c_2 for $j(i)$ on SU-BS link in the second slot (*i.e.* $w^{c_2}_{j(i),P} = 0.5\log(1+2\bar{p}_{j(i)} g^{c_2}_{j(i),P})/(\bar{R}_i(t) - \bar{R}^{\min}_i(t))$. $w^{c_2}_j$ denotes the normalized marginal utility of SU j on being assigned c_2 for its own data $0.5\log(1+2\bar{p}_j g^{c_2}_j)/\bar{R}_j(t)$, and a_i, b_j denote the aggregate marginal utility (flow) achieved by PU i and SU j respectively. $x^{c_1}_i$ is the binary variable denoting whether c_1 is assigned to PU i in the first time slot, $y^{c_2}_i$ denotes whether c_2 is assigned to i's helper SU $j(i)$ for relaying in the second time slot, and $y^{c_2}_j$ denotes whether c_2 is assigned to SU j for its own transmission in the second time slot.

$$\max_{x^{c_1}_i, y^{c_2}_i, y^{c_2}_j} \sum_{i\in\mathcal{N}^{\mathcal{R}}_{\mathcal{P}}} a_i + \sum_{j=N_P+1}^{N} b_j \tag{19}$$

$$\text{s.t.} \sum_{c_2\in\mathcal{K}^{\mathcal{R}}} y^{c_2}_j \cdot w^{c_2}_j = b_j, \forall j \in [N_P+1, N],$$

$$\sum_{c_1\in\mathcal{K}^{\mathcal{R}}} x^{c_1}_i \cdot w^{c_1}_{i,j(i)} = a_i, \sum_{c_2\in\mathcal{K}^{\mathcal{R}}} y^{c_2}_i \cdot w^{c_2}_{j(i),P} = a_i, \forall i \in \mathcal{N}^{\mathcal{R}}_{\mathcal{P}},$$

$$\sum_{i\in\mathcal{N}^{\mathcal{R}}_{\mathcal{P}}} x^{c_1}_i = 1, \sum_{i\in\mathcal{N}^{\mathcal{R}}_{\mathcal{P}}} y^{c_2}_i + \sum_{j\in[N_P+1,N]} y^{c_2}_j = 1, \forall c_1, c_2 \in \mathcal{K}^{\mathcal{R}},$$

Theorem 3: The subcarrier assignment problem under the above IP formulation is NP-hard.

Proof: The problem can be reduced from type-dependent *multiple knapsack problems* (MKP), where each set of knapsacks (users) belongs to a different type (time slot and primary/secondary). The profit of allocating an item (subcarrier) depends not only on the knapsacks but also the type of them. The one-type MKP is known to be NP-hard and even hard to approximate [16]. Therefore our problem is NP-hard. ■

Given the hardness of the problem, we present a rounding based algorithm to solve it as shown in **Algorithm 2**. It ensures that each subcarrier is assigned to at most one user for both slots. We now capture the performance of the algorithm.

Theorem 4: **Algorithm 2** provides an approximation guarantee of at least $1 - \sqrt{\frac{4cN_S}{K^R}\ln(K^R)}$ with high probability, where K^R is the cardinality of the subcarrier set $\mathcal{K}^{\mathcal{R}}$.

Proof: Refer to the Appendix for a detailed proof. ■

Therefore, its performance becomes better when there is a larger magnitude of available subcarriers to users in the system. Since the number of subcarriers in a practical OFDMA system is much bigger than that of users, **Algorithm 2** can be expected to provide good performance.

D. Power Control

After all the subcarriers are allocated as above, power can be allocated to each user optimally. For PUs with direct transmission, optimal power allocation is a simple water-filling solution. For PUs with cooperative transmission, optimal power allocation is performed on a per-pair basis with their unique helping SUs. With subcarriers allocated and their use on an SU determined, power allocation on each pair of PU-SU is a standard convex optimization problem and can be readily solved by KKT conditions. We omit the details here.

Algorithm 2 *Rounding-based Subcarrier Allocation*

1. Formulate the problem using the IP above. Solve its LP relaxation with $x^{c_1}_i, y^{c_2}_i, y^{c_2}_j$ being relaxed to $[0,1]$. Let the LP solutions be $\hat{x}^{c_1}_i, \hat{y}^{c_2}_i, \hat{y}^{c_2}_j$ and $\hat{a}_i, \hat{b}_j$.
2. Adopt the following procedure to round the fractional solutions, $\hat{x}^{c_1}_i, \hat{y}^{c_2}_n$, to integral values, $\tilde{x}^{c_1}_i, \tilde{y}^{c_2}_n$, where $n \in \{i \in \mathcal{N}^{\mathcal{R}}_{\mathcal{P}}\} \cup \{j \in [N_P+1, N]\}$.
   - For every c_2, round $y^{c_2}_n$ to 1 ($\tilde{y}^{c_2}_n$) with probability $\hat{y}^{c_2}_n$. If $\tilde{n}$ is the user to whom c_2 is assigned, then $\hat{y}^{c_2}_n = 0, \forall n \neq \tilde{n}$.
   - Update $\tilde{a}_i = \frac{\sum \tilde{y}^{c_2}_i w^{c_2}_{j(i),P}}{1-\delta}, \tilde{b}_j = \frac{\sum \tilde{y}^{c_2}_j w^{c_2}_j}{1-\delta}$, where δ is a constant derived below. Run the LP again on $x^{c_1}_i$ only. Let $\bar{x}^{c_1}_i$ be the solutions of the new LP.
   - For c_1, round $x^{c_1}_i$ to 1 ($\tilde{x}^{c_1}_i$) with probability $\bar{x}^{c_1}_i$. If $\tilde{i}$ is the PU c_1 is assigned to, then $\tilde{x}^{c_1}_i = 0, \forall i \neq \tilde{i}$.

V. Performance Evaluation

To evaluate the performance of FLEC with proposed algorithms, we adopt empirical parameters to model the fading environment. There are 128 subcarriers centered at 2.5 GHz with bandwidth 312.5 kHz. Channel gain can be decomposed into a large-scale log normal shadowing component with standard deviation of 5.8 and path loss exponent of 4, and a small-scale Rayleigh fading component. The inherent frequency selectivity is captured by an exponential power delay profile with delay spread 1.257 μs as reported via extensive measurements [17]. The entire 40 MHz channel is partitioned into blocks of size equal to the coherence bandwidth $B_c \approx 795.6$ KHz. Three independent Rayleigh waveforms are generated for each block using the modified Jakes fading model and a weighted sum is taken to calculate the SNR. A scheduling epoch is of 5 ms duration, and an evaluation period consists of 1000 scheduling epochs. The number of PUs is set to 60, and the number of SUs varies. Such setup is commonly used in related works [9].

A. Overall performance of FLEC

We first evaluate the overall performance of distributed and centralized FLEC compared with conventional identical channel cooperation ("ICC" in the figures). Resource allocation of ICC can be similarly formulated as that of FLEC, with per-subcarrier flow conservation constraints instead of total flow conservation (4), and our algorithms are readily applicable with minor adjustments. We then apply a revised *Centralized Optimization* to derive the *optimal* ICC performance as the benchmark here. In Fig. 2, we plot the average throughput of both PUs (first three bars) and SUs (last three bars). We can see that distributed and centralized FLEC, implemented with *Distributed Bargaining* and *Centralized Heuristic* as in Sec. III-IV provide 20–40% and 30–60% improvement, respectively. It clearly demonstrates the advantage of FLEC. A similar trend is also observed for SUs,

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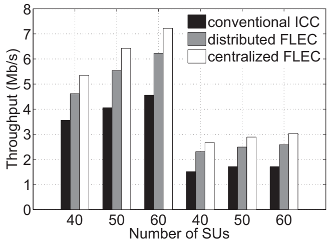

Fig. 2. Overall throughput performances of the proposed algorithms.

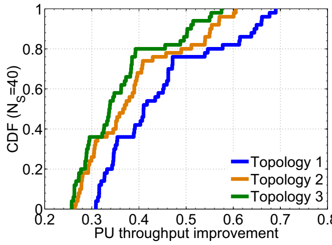

Fig. 3. Effects of topology on PU throughput improvement.

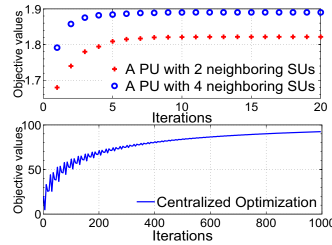

Fig. 4. Convergence of the proposed algorithms ($N_S = 50$).

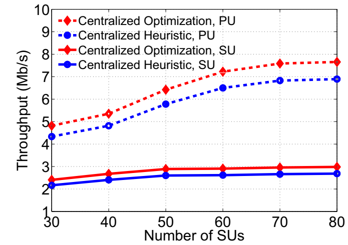

Fig. 5. Near-optimal performance of *Centralized Heuristic*.

although the improvement becomes marginal when the number of SUs scales up. The reason is that, though a larger number of SUs provides more and better cooperation for PUs and thus improves their throughput, it results in fewer channels leased to each SU, and a lower degree of optimization freedom.

B. Effects of Topology

Next we investigate the effects of topology on FLEC. We choose *Distributed Bargaining* as the representative algorithm, and evaluate three representative topologies, where N_S equals 40 and the average distance from PU to BS is controlled to be 0.8, 0.65, and 0.5 of the cell radius (topology 1, 2, and 3 respectively). We observe from Fig. 3 that while cooperation always results in some improvement in PU throughput, scenarios dominated by high path loss and poor shadowing benefit the most (topology 1), as more cooperation opportunities can be explored. SU's throughput also becomes better in these scenarios, which we do not show here due to space limit. This observation justifies the deployment of SU cooperation for throughput enhancement in primary networks with high path loss and limited coverage.

C. Convergence of FLEC algorithms

We study convergence of the algorithms proposed to realize FLEC, which affects their practicality. Fig. 4 shows the convergence of *Distributed Bargaining* for two randomly chosen PUs with different number of neighboring SUs. It is clear that *Distributed Bargaining* converges within 20 iterations, validating its feasibility in practice. The reason for the fast convergence, as discussed in Sec. III-D, is mainly the limited size of neighborhood. With distributed and concurrent operations, it is indeed suitable for practical implementation.

We also observe that *Centralized Optimization* does not converge even after 1000 iterations in Fig. 4. This echos our concern about the complexity of centralized subgradient update of two vector dual variables in Sec. IV, and justifies our motivation to design efficient heuristics.

D. Near-optimality of *Centralized Heuristic*

We now evaluate the performance loss of the sub-optimal *Centralized Heuristic* compared with that of *Centralized Optimization*. As seen from Fig. 5, with respect to the average throughput of both PU and SU, *Centralized Heuristic* losses about 5% in all cases. Due to the slow convergence of *Centralized Optimization*, we may conclude that *Centralized Heuristic* achieves a good tradeoff between performance and complexity, and is amenable to practical implementations.

To further understand the practicality of *Centralized Heuristic*, we observe the running time of its component algorithms in our simulations. We find that all of them are of the order of milliseconds on an Intel Xeon Quad-core CPU running at 3GHz with 2GB memory. Therefore usual scheduling deadlines of 5–10 ms [3] can be satisfactorily met. Due to space limit, we omit the data here.

VI. Related Work

A plethora of works has been done on spectrum sharing based on cognitive radio. Generally, they fall into three paradigms, *interweave, underlay*, and *overlay* [18]. The interweave paradigm insists that SUs should only transmit when PUs are not, while the underlay paradigm allows them to transmit concurrently with PUs provided that their signals do not cause harmful interference. Essentially, in both cases SUs are transparent to PUs. The *overlay* paradigm, which is the focus of this paper, assumes PUs have side information about SUs, and leverage them to improve primary network performance. However, most existing works on either information theoretic analysis [19] or implementation issues [2] adopt a single-channel setting. In contrary, we consider a multi-channel setting with multiple PUs and SUs, and seek to provide a practical framework to implement cooperative cognitive radio networks.

Resource allocation in cognitive radio and cooperative networks have been extensively studied. For the former, most works [20], [21] consider maximizing SUs' throughput with constrained interference to PUs. For the latter, [9] and our previous work [10] address the problem with different primary user cooperation schemes. In a multi-channel CCRN, primary and secondary networks have to be *jointly* considered, aggravating the optimization problem. Moreover, existing works invariantly assume that relays are obedient and altruistic to help, as they do not possess their own data to transmit. In CCRN, PUs and SUs are selfish, and fairness between them needs to be addressed appropriately for cooperation to take place. We apply the concepts of Nash Bargaining Solutions [6] to ensure both parities benefit from cooperation, and are not taken advantage of unfairly. NBS has been applied to allocate resources in cooperative OFDMA networks [7], [8]. These works do not consider the inefficiency of conventional cooperation methods in the context of multi-channel CCRN, and only heuristics without any performance bounds are given.

VII. Concluding Remarks

This work represents an early attempt to study multi-channel cooperative cognitive radio networks. The central question addressed in the paper is how to effectively exploit secondary user cooperation when conventional cooperation method becomes inefficient in this scenario, which has not yet been explored. We propose FLEC, a flexible channel cooperation design to allow SUs to customize the use of leased resources in order to maximize performance. We develop a unifying optimization framework based on Nash Bargaining Solutions to address the resource allocation problem with FLEC, where relay assignment, subcarrier assignment, relay strategy optimization and power control intricately interplay with one another. An optimal distributed algorithm as well as an efficient centralized heuristic with near-optimal performance are proposed. Simulation studies corroborate that FLEC is efficient and improves the performance of both primary and secondary networks.

Appendix

Proof of Theorem 4: The proof is based on the Chernoff Bound. For $v_i \in [0,1]$ independent random variables, let $S = \sum_i v_i, \mu = E[\sum_i v_i]$, then

$$P[S \leq (1-\delta)\mu] \leq e^{-\frac{\delta^2 \mu}{2}}.$$

Let $\Phi_i^1 = \sum_{c_1} \tilde{x}_i^{c_1} w_{i,j(i)}^{c_1}, \Phi_i^2 = \sum_{c_2} \tilde{y}_i^{c_2} w_{j(i),P}^{c_2}, \Phi_j^2 = \sum_{c_2} \tilde{y}_j^{c_2} w_j^{c_2}$. All $\Phi_i^1, \Phi_i^2, \Phi_j^2$ are sums of R.V's $\in [0,1]$. With $E[\Phi_i^2] = \hat{a}_i, E[\Phi_j^2] = \hat{b}_j$, we have the following

$$P[\Phi_i^2 \geq (1-\delta)\hat{a}_i] \geq 1-e^{-\frac{\delta^2 \hat{a}_i}{2}}, P[\Phi_j^2 \geq (1-\delta)\hat{b}_j] \geq 1-e^{-\frac{\delta^2 \hat{b}_j}{2}}.$$

Note that the aggregate marginal utility (flow) in the first time slot is obtained from the flow in the rounded solution in the second slot. Assume $\sum_{c_2} \tilde{y}_i^{c_2} w_{j(i),P}^{c_2} \geq (1-\delta)\hat{a}_i$. Since $E[\Phi_i^1 | \Phi_i^2] = \tilde{a_i} = \Phi_i^2/(1-\delta) \geq \hat{a}_i$, it can be shown that

$$P[\Phi_i^1 \geq (1-\delta)\hat{a}_i | \Phi_i^2] \geq 1 - e^{-\frac{\delta^2 \hat{a}_i}{2}}.$$

Now to ensure a net flow of at least $(1-\delta)\hat{a}_i$ at every PU and $(1-\delta)\hat{b}_j$ at every SU with high probability, we need:

$$P[\Phi_i^1, \Phi_i^2 \geq (1-\delta)\hat{a}_i] \geq \left(1 - e^{-\frac{\delta^2 \hat{a}_i}{2}}\right)^2 = \left(1 - \frac{1}{K^R}\right)^2,$$

$$P[\Phi_j^2 \geq (1-\delta)\hat{b}_j] \geq \left(1 - e^{-\frac{\delta^2 \hat{b}_j}{2}}\right)^2 = \left(1 - \frac{1}{K^R}\right)^2.$$

This results in $\delta = \sqrt{\frac{2\ln(K^R)}{\hat{a}_i}} = \sqrt{\frac{2\ln(K^R)}{\hat{b}_j}}$. Then the approximation bound B is given by

$$B = 1 - \sqrt{2\ln(K^R)} \frac{\sum_i \sqrt{\hat{a}_i} + \sum_j \sqrt{\hat{b}_j}}{\sum_i \hat{a}_i + \sum_j \hat{b}_j} \tag{20}$$

$\frac{\sum_i \sqrt{\hat{a}_i} + \sum_j \sqrt{\hat{b}_j}}{\sum_i \hat{a}_i + \sum_j \hat{b}_j}$ is maximum when $\hat{a}_i = \hat{b}_j = \hat{a} = \frac{\sum_i \hat{a}_i + \sum_j \hat{b}_j}{2N_S}$. By the normalized weight assumption $\sum_i \hat{a}_i + \sum_j \hat{b}_j \leq K^R$. If $c = \frac{\max_i \{w_{i,j(i)}^{c1}, w_{j(i),P}^{c2}\}}{\min_{i,j} \{w_{i,j(i)}^{c1}, w_{j(i),P}^{c2}\}} \geq 1$, then $\sum_i \hat{a}_i + \sum_j \hat{b}_j \geq \frac{K^R}{c}$. Substituting into (20), we have the following with high probability

$$B \geq 1 - \sqrt{\frac{4cN_S}{K^R} \ln(K^R)}.$$

■

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